Analysis by Default

(Days 2 and 3)

*** Working draft ***

Version of: July 7, 2016

Outline

1. Common law constraint

   Contested lexical items in the law
   Common law constraint
   A formal model of constraint
   Constraining meaning

2. Default logic

   Prioritized default theories
   Variable priorities and undercutting

3. Coding constraint into default logic

   A direct encoding
   An encoding into variable priority theories

4. Elaborations

   A fortiori constraint
   Dimensions and magnitudes
   Boolean definitions

Part 1.
Common Law Constraint

Some contested lexical items

- "Sandwich"

  Is a burrito a sandwich?

- "Potato chip"

  Are Pringles potato chips?

- "Vessel"

  Is the Super Scoop a Vessel?

- "Employee"

  Are Uber drivers employees, or contractors?

- "Trade secret"

  Is the Lynchburg Lemonade recipe a trade secret?

Constraint in the common law

1. Several approaches:

    - There is no such thing

    - Coherence accounts

    - Constraint depends on rules

        - The rules are defeasible

        - The rules are strict

            Rules must be applied as stated

            Distinguishing allowed

2. Distinguishing:

    Identifying differences between facts of precedent and current cases, to explain why precedent rule should not be applied to current case, with the result that . . .

    . . . the rule is modified

3. Example:

Case 1: Can Emma watch TV?

Facts: age 9, no dinner, did homework

Rule: at least 9 $\rightarrow$ TV

Outcome: TV

Case 2: Can Max watch TV?

Facts: age 14, no dinner, no homework

Rule: no homework $\rightarrow$ no TV

Outcome: no TV

4. A problem:

How can a court be constrained by rules, if it is able to modify those rules at will?

5. A solution (Raz/Simpson):

   Courts can modify previous rules, but not entirely at will—subject to two conditions:

   - Can only narrow previous rules, by adding further restrictions (that distinguish that case from this one)

   - The narrowed rules must continue to yield same results in the earlier cases

6. A further problem:

   Why suppose that courts can modify rules in exactly *this* way?

7. My goal:

   Answer this question . . .

   But then the answer suggests a different view of common law reasoning

Factors, reasons, rules, cases

1. Factors for π and δ:

$$F^\pi = \{f_1^\pi, \ldots, f_n^\pi\}$$
$$F^\delta = \{f_1^\delta, \ldots, f_m^\delta\}$$
$$F = F^\pi \cup F^\delta$$

2. Examples:

   In domentic domain:

   Age 9 or older
   Didn't eat dinner
   Hit sister
   Had a bad dentist visit

   In trade secrets doman (Rissland, Ashley):

   Took measures to protect information
   Confidentiality agreement
   Information publicly available
   Information reverse-engineerable

3. Fact situation:

$$X \subseteq F$$
$$X^\pi = X \cap F^\pi$$
$$X^\delta = X \cap F^\delta$$

4. (Factor) reason: $X \subseteq F^s$, where s is π or δ

   Example: $\{f_1^\pi, f_2^\pi\}$ is a reason, $\{f_1^\pi, f_3^\delta\}$ is not

5. When a reason holds:

$$X \models R \text{ iff } R \subseteq X$$
$$X \models \neg\phi \text{ iff } X \not\models \phi$$
$$X \models \phi \wedge \psi \text{ iff } X \models \phi \text{ and } X \models \psi$$

   Example: $\{f_1^\pi, f_3^\pi, f_2^\delta, f_3^\delta\} \models \{f_1^\pi, f_3^\pi\} \wedge \neg\{f_1^\delta\}$

6. Rule: Where R^s is a reason for s and $R_1^{\overline{s}}, \ldots R_i^{\overline{s}}$ are reasons for $\overline{s}$, a rule for s has form:

$$R^s \wedge \neg R_1^{\overline{s}} \wedge \ldots \wedge \neg R_i^{\overline{s}} \rightarrow s$$

   (R^s would be the *reason* for the decision)

   Example:

$$\{f_1^\pi, f_3^\pi\} \wedge \neg\{f_1^\delta\} \wedge \neg\{f_2^\delta, f_4^\delta\} \rightarrow \pi$$

   Some housekeeping functions:

$$Prem(r) = R^s \wedge \neg R_1^{\overline{s}} \wedge \ldots \wedge \neg R_i^{\overline{s}}$$
$$Prem^s(r) = R^s$$
$$Conc(r) = s$$

7. Case: $c = \langle X, r, s \rangle$, where

$$\mathit{Facts}(c) = X$$
$$\mathit{Rule}(c) = r$$
$$\mathit{Outcome}(c) = s$$

subject to condition that

$$X \models \mathit{Prem}(r)$$

8. Example: $c_1 = \langle X_1, r_1, s_1 \rangle$, where

$$X_1 = \{f_1^\pi, f_2^\pi, f_3^\pi, f_1^\delta, f_2^\delta, f_3^\delta, f_4^\delta\}$$
$$r_1 = \{f_1^\pi, f_2^\pi\} \wedge \neg\{f_1^\delta, f_5^\delta\} \rightarrow \pi$$
$$s_1 = \pi$$

9. A case base Γ is a set of cases

The reason model

1. Consider $c_2 = \langle X_2, r_2, s_2 \rangle$, with

$$X_2 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$$
$$r_2 = \{f_1^\pi\} \rightarrow \pi$$
$$s_2 = \pi$$

What is the court telling us with c_2 ?

Two things:

- $Prem^\pi(r_2) = \{f_1^\pi\}$ is a sufficient reason for π
- $Prem^\pi(r_2)$ is stronger than the strongest reason present for δ

The strongest reason present for δ is:

$$X_2^\delta = \{f_1^\delta, f_2^\delta\}$$

Therefore:

$$X_2^\delta <_{c_2} Prem^\pi(r_2)$$

or

$$\{f_1^\delta, f_2^\delta\} <_{c_2} \{f_1^\pi\}$$

2. Continue with $c_2 = \langle X_2, r_2, s_2 \rangle$, with

$$X_2 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$$
$$r_2 = \{f_1^\pi\} \to \pi$$
$$s_2 = \pi$$

So we have $X_2^\delta <_{c_2} \mathit{Prem}^\pi(r_2)$ — anything else?

Yes. If W weaker than X_2^δ and Z is stronger than $\mathit{Prem}^\pi(r_2)$, we have:

$$W <_{c_2} Z$$

Example: since

$$\{f_1^\delta, f_2^\delta\} <_{c_2} \{f_1^\pi\}$$

we have

$$\{f_1^\delta\} <_{c_2} \{f_1^\pi, f_4^\pi\}$$

3. Preference derived from a case $c = \langle X, r, s \rangle$:

$W <_c Z$ iff $W \subseteq X^{\overline{s}}$ and $\mathit{Prem}^s(r) \subseteq Z$

4. Preference derived from a case base Γ:

$$W <_\Gamma Z \text{ iff there is } c \text{ in } \Gamma \text{ such that } W <_c Z$$

5. The case base Γ is inconsistent iff there are reasons X and Y such that

$$X <_\Gamma Y \quad \text{and} \quad Y <_\Gamma X$$

   The case base is consistent iff it is not inconsistent

6. The reason model of constraint:

   > Given Γ and new fact situation X, the reason model of constraint requires a decision based on some rule r for outcome s such that $\Gamma \cup \{\langle X, r, s\rangle\}$ consistent.

7. Example: $\Gamma = \{c_2\}$, where

$c_2 = \langle X_2, r_2, s_2 \rangle$, with

$$X_2 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$$

$$r_2 = \{f_1^\pi\} \rightarrow \pi$$

$$s_2 = \pi$$

New fact situation

$$X_3 = \{f_1^\pi, f_1^\delta, f_2^\delta, f_3^\delta\}$$

Suppose court wants to decide this case for δ on the basis of $\{f_1^\delta, f_2^\delta\}$, leading to

$c_3 = \langle X_3, r_3, s_3 \rangle$, with

$$X_3 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$$

$$r_3 = \{f_1^\delta, f_2^\delta\} \rightarrow \delta$$

$$s_3 = \delta$$

Then new case base is $\Gamma' = \{c_2, c_3\}$, but this is inconsistent:

$$\{f_1^\delta, f_2^\delta\} <_{c_2} \{f_1^\pi\}$$
$$\{f_1^\pi\} <_{c_3} \{f_1^\delta, f_2^\delta\}$$

and both $c_2, c_3 \in \Gamma'$

So decision ruled out by reason constraint

8. Another path: $\Gamma = \{c_2\}$ again, where

$c_2 = \langle X_2, r_2, s_2 \rangle$, with

$$X_2 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$$

$$r_2 = \{f_1^\pi\} \rightarrow \pi$$

$$s_2 = \pi$$

New fact situation

$$X_4 = X_3 = \{f_1^\pi, f_1^\delta, f_2^\delta, f_3^\delta\}$$

Now decide this case for δ on the basis of $\{f_1^\delta, f_3^\delta\}$, leading to $\Gamma = \{c_2, c_4\}$, where

$c_4 = \langle X_4, r_4, s_4 \rangle$, with

$$X_4 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$$

$$r_4 = \{f_1^\delta, f_3^\delta\} \rightarrow \delta$$

$$s_4 = \delta$$

This case base is consistent, with constraints

$$\{f_1^\delta, f_2^\delta\} <_{c_2} \{f_1^\pi\}$$

$$\{f_1^\pi\} <_{c_4} \{f_1^\delta, f_3^\delta\}$$

9. Hypothesis: this is how case law develops—by building up a stronger and stronger ordering on reasons

10. Now: two models of precedential constraint

Standard model:

- What's important is rules
- Constrained to make decisions that can be accommodated by rule modification, in accord with Raz/Simpson conditions
- As law develops, rules become more complicated

Reason model:

- What's important is ordering relation on reasons
- Constrained to make decisions consistent with this ordering
- As law develops, ordering becomes stronger

What is the relation between them?

11. Central result (equivalence):

Given Γ and a new situation X, a decision based on some rule r favoring s satisfies the standard model of constraint iff it satisfied the reason model of constraint

Quasi-technical issues

1. Preference ordering is not transitive. Consider $\Gamma = \{c_5, c_6, c_7\}$, where

   $c_5 = \langle X_5, r_5, s_5 \rangle$, with

   $$X_5 = \{f_1^\pi, f_1^\delta\}$$
   $$r_5 = \{f_1^\pi\} \rightarrow \pi$$
   $$s_5 = \pi$$

   $c_6 = \langle X_6, r_6, s_6 \rangle$, with

   $$X_6 = \{f_1^\pi, f_2^\delta\}$$
   $$r_6 = \{f_2^\delta\} \rightarrow \delta$$
   $$s_6 = \delta$$

   $c_7 = \langle X_7, r_7, s_7 \rangle$, with

   $$X_7 = \{f_2^\pi, f_2^\delta\}$$
   $$r_7 = \{f_2^\pi\} \rightarrow \pi$$
   $$s_7 = \pi$$

   Then have

   $$\{f_1^\delta\} <_\Gamma \{f_1^\pi\} <_\Gamma \{f_2^\delta\} <_\Gamma \{f_2^\pi\}$$

   But not

   $$\{f_1^\delta\} <_\Gamma \{f_2^\pi\}$$

Solution: replace $<_\Gamma$ with its transitive closure in definition of inconsistency

Old version:

The case base Γ is consistent iff there are no reasons X and Y such that

$$X <_\Gamma Y \quad \text{and} \quad Y <_\Gamma X$$

New version:

The case base Γ is consistent iff there are no reasons X and Y such that

$$X \prec_\Gamma Y \quad \text{and} \quad Y \prec_\Gamma X$$

where $\prec_\Gamma$ is the transitive closure of $<_\Gamma$

Question: do we want the solution??

2. Our definitions assume consistency of background case base. But even it it's inconsistent, we can modify definitions to require that new decisions introduce "no more" inconsistency

Old version:

> Given Γ and new fact situation X, the reason model of constraint requires a decision based on some rule r for outcome s such that $\Gamma \cup \{\langle X, r, s \rangle\}$ consistent.

New version:

> Given Γ and new fact situation X, the reason model of constraint requires a decision based on some rule r for outcome s such that: whenever $Y <_{\Gamma \cup \{\langle X,r,s \rangle\}} Z$ and $Z <_{\Gamma \cup \{\langle X,r,s \rangle\}} Y$, we also have $Y <_{\Gamma} Z$ and $Z <_{\Gamma} Y$.

3. The law is not the rules

Example: recall $\Gamma = \{c_2\}$, where

$c_2 = \langle X_2, r_2, s_2 \rangle$, with

$$X_2 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$$

$$r_2 = \{f_1^\pi\} \to \pi$$

$$s_2 = \pi$$

Recall that

$$X_3 = \{f_1^\pi, f_1^\delta, f_2^\delta, f_3^\delta\}$$

could be decided for δ on the basis of $\{f_1^\delta, f_3^\delta\}$, leading to $\Gamma = \{c_2, c_3\}$, where

$c_3 = \langle X_3, r_3, s_3 \rangle$, with

$$X_3 = \{f_1^\pi, f_1^\delta, f_2^\delta, f_3^\delta, \}$$

$$r_3 = \{f_1^\delta, f_3^\delta\} \to \delta$$

$$s_3 = \delta$$

and so imposing constraints

$$\{f_1^\delta, f_2^\delta\} <_{c_2} \{f_1^\pi\}$$
$$\{f_1^\pi\} <_{c_3} \{f_1^\delta, f_3^\delta\}$$

But suppose that, prior to facing X_3, the court confronted the situation

$$X_8 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_3^\delta\}$$

and decided for π through an application of the rule r_2, leading to $\Gamma = \{c_2, c_8\}$, where

$c_8 = \langle X_8, r_8, s_8 \rangle$, with

$$X_8 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_3^\delta\}$$

$$r_8 = \{f_1^\pi\} \rightarrow \pi$$

$$s_8 = \pi$$

so imposing the constraint

$$\{f_1^\delta, f_3^\delta\} <_{c_8} \{f_1^\pi\}$$

But with this new constraint, the c_3 decision is no longer possible

Upshot: the sequence

$$c_2, c_3$$

is allowable, but the sequence

$$c_2, c_8, c_3$$

is not, even though c_8 results simply from an application of the c_2 rule

Semantics of the contested lexicon

1. Idea: Same theory, but instead of

$$\pi, \delta$$

focus on

$$v, \overline{v}$$

where v is some contested lexical item

2. Example (Super Scoop):

$$v = \text{vessel}$$

with factors

f_1^v = subject to Coast Guard regulations
f_2^v = captain and crew
f_3^v = navigation lights
f_4^v = ballast tanks
f_5^v = galley for crew

$f_1^{\overline{v}}$ = no self-propulsion
$f_2^{\overline{v}}$ = primary business not navigation
$f_3^{\overline{v}}$ = not moving at time

Previous case (the Betty F):

$c_9 = \langle X_9, r_9, s_9 \rangle$, with

$$X_9 = \{f_1^v, f_2^v, f_3^v, f_4^v, f_1^{\overline{v}}, f_2^{\overline{v}}, f_3^{\overline{v}}\}$$

$$r_9 = \{f_2^{\overline{v}}, f_3^{\overline{v}}\} \rightarrow \overline{v}$$

$$s_9 = \overline{v}$$

So $\Gamma = \{c_9\}$ and have

$$\{f_1^v, f_2^v, f_3^v, f_4^v\} <_\Gamma \{f_2^{\overline{v}}, f_3^{\overline{v}}\}$$

New fact situation (the Super Scoop):

$$X_{10} = \{f_1^v, f_2^v, f_3^v, f_4^v, f_2^{\overline{v}}, f_3^{\overline{v}}\}$$

Court constrained to decide for $\overline{v}$, leading to $\Gamma' = \{c_9, c_{10}\}$ with

$c_{10} = \langle X_{10}, r_{10}, s_{10} \rangle$, with

$$X_{10} = \{f_1^v, f_2^v, f_3^v, f_4^v, f_2^{\overline{v}}, f_3^{\overline{v}}\}$$

$$r_{10} = \{f_2^{\overline{v}}, f_3^{\overline{v}}\} \rightarrow \overline{v}$$

$$s_{10} = \overline{v}$$

3. Example (Super Scoop, modified):

As before $\Gamma = \{c_9\}$ with

$c_9 = \langle X_9, r_9, s_9 \rangle$, with

$$X_9 = \{f_1^v, f_2^v, f_3^v, f_4^v, f_1^{\overline{v}}, f_2^{\overline{v}}, f_3^{\overline{v}}\}$$
$$r_9 = \{f_2^{\overline{v}}, f_3^{\overline{v}}\} \rightarrow \overline{v}$$
$$s_9 = \overline{v}$$

So

$$\{f_1^v, f_2^v, f_3^v, f_4^v\} <_\Gamma \{f_2^{\overline{v}}, f_3^{\overline{v}}\}$$

Imagine (modified Super Scoop):

$$X'_{10} = \{f_1^v, f_2^v, f_3^v, f_4^v, f_5^v, f_2^{\overline{v}}, f_3^{\overline{v}}\}$$

Now court *can* decide for v on basis, say, of

$$\{f_5^v\} \rightarrow v$$

leading to

$$\{f_2^{\overline{v}}, f_3^{\overline{v}}\} < \{f_5^v\}$$

But is that sensible??

Part 2.
Default Logic

Fixed priority default theories

1. Example:

   Tweety is a bird
   Therefore, Tweety is able to fly

   Why? There is a default that birds fly

   Tweety is a penguin
   Therefore, Tweety is not able to fly

   Because there is a (stronger) default that penguins don't fly

2. Another example:

   I promised to meet Ann for lunch
   Therefore, I ought to meet Ann for lunch

   Why? I should do what I promise, by default

   I see a drowning child
   Therefore, I ought to rescue the child
   (and so) not meet Ann for lunch

   Because there is a (stronger) default that favors rescuing the child

3. Default rules: $X \rightarrow Y$

   Example: $B(t) \rightarrow F(t)$

   Instance of: $B(x) \rightarrow F(x)$ ("Birds fly")

4. Premise and conclusion:

   If $\delta = X \rightarrow Y$, then

$$Prem(\delta) = X$$
$$Conc(\delta) = Y$$

   If $\mathcal{D}$ set of defaults, then

$$Conc(\mathcal{D}) = \{Conc(\delta) : \delta \in \mathcal{D}\}$$

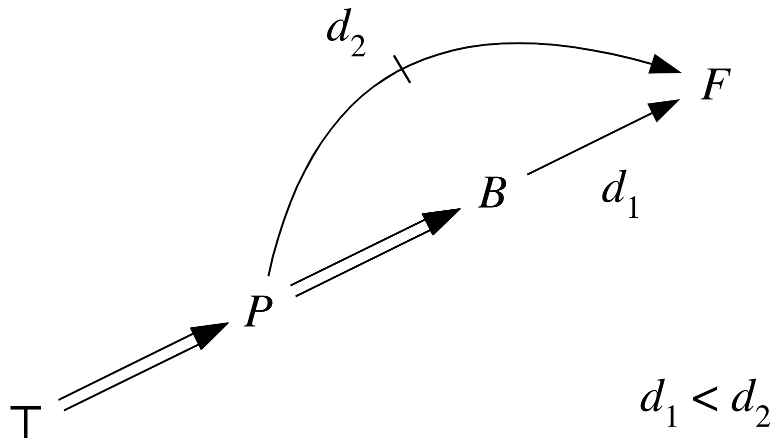
5. Priority ordering on defaults (strict, partial)

   $\delta < \delta'$ means: δ' stronger than δ

6. Priorities have different sources:

   Specificity
   Reliability
   Authority
   Our own reasoning

   For now, take priorities as fixed, leading to . . .



7. A *fixed priority default theory* is a tuple

$$\langle \mathcal{W}, \mathcal{D}, < \rangle$$

where $\mathcal{W}$ contains ordinary statements, $\mathcal{D}$ contains defaults, and $<$ is an ordering

8. Example (Tweety Triangle):

$$\begin{array}{lll} \mathcal{W} & = & \{P, P \Rightarrow B\} \\ \mathcal{D} & = & \{\delta_1, \delta_2\} \\ \delta_1 & = & B \rightarrow F \\ \delta_2 & = & P \rightarrow \neg F \\ \delta_1 < \delta_2 & & \end{array}$$

9. Another example (Drowning child):

$$\begin{array}{lll} \mathcal{W} & = & \{P, D, \neg(M \wedge R)\} \\ \mathcal{D} & = & \{\delta_1, \delta_2\} \\ \delta_1 & = & P \rightarrow M \\ \delta_2 & = & D \rightarrow R \\ \delta_1 < \delta_2 & & \end{array}$$

10. Main question: what can we conclude from such a theory?

11. An *extension* $\mathcal{E}$ of $\langle \mathcal{W}, \mathcal{D}, < \rangle$ is a belief set an ideal reasoner might settle on, based this information

    Usually defined directly, but we take roundabout route . . .

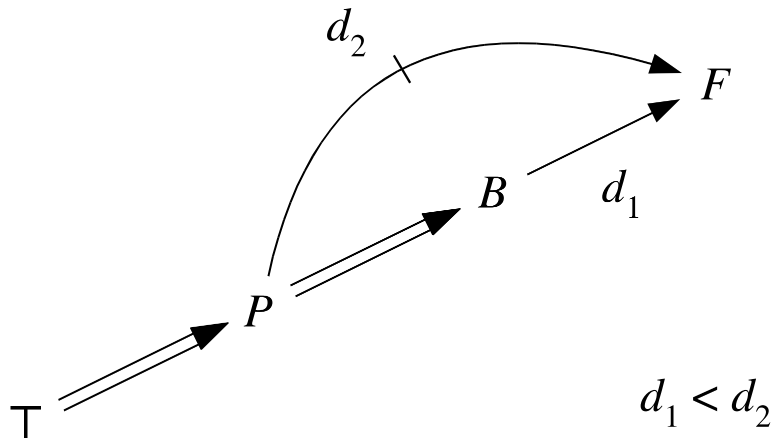
12. A *scenario* based on $\langle \mathcal{W}, \mathcal{D}, < \rangle$ is some subset $\mathcal{S}$ of the defaults $\mathcal{D}$

13. A *proper scenario* is the "right" subset of defaults

14. An *extension* $\mathcal{E}$ based on $\langle \mathcal{W}, \mathcal{D}, < \rangle$ is a set

    $$\mathcal{E} = Th(\mathcal{W} \cup Conc(\mathcal{S}))$$

    where $\mathcal{S}$ is a proper scenario



15. Returning to example: $\langle \mathcal{W}, \mathcal{D}, < \rangle$ where

$$
\begin{array}{lcl}
\mathcal{W} & = & \{P, P \Rightarrow B\} \\
\mathcal{D} & = & \{\delta_1, \delta_2\} \\
\delta_1 & = & B \rightarrow F \\
\delta_2 & = & P \rightarrow \neg F \\
\delta_1 < \delta_2 & &
\end{array}
$$

Four possible scenarios:

$$
\begin{array}{l}
\mathcal{S}_1 = \emptyset \\
\mathcal{S}_2 = \{\delta_1\} \\
\mathcal{S}_3 = \{\delta_2\} \\
\mathcal{S}_4 = \{\delta_1, \delta_2\}
\end{array}
$$

But only $\mathcal{S}_3$ proper ("right"), so extension is

$$
\begin{array}{lcl}
\mathcal{E}_3 & = & Th(\mathcal{W} \cup Conc(\mathcal{S}_3)) \\
 & = & Th(\{P, P \supset B\} \cup \{\neg F\}) \\
 & = & Th(\{P, P \supset B, \neg F\}),
\end{array}
$$

16. Immediate goal: specify *proper scenarios*

Binding defaults

1. Defined through preliminary concepts:

   Triggering

   Conflict

   Defeat

2. Triggered defaults:

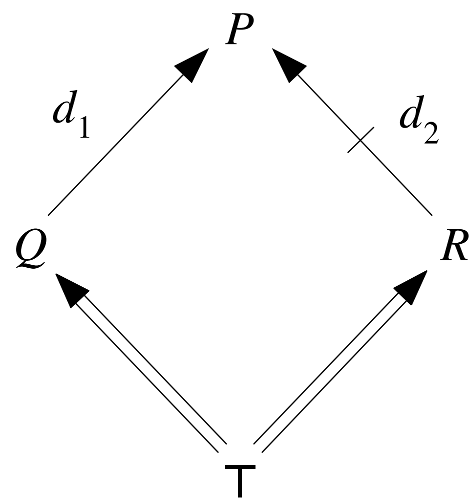
   $$\mathit{Triggered}_{\mathcal{W},\mathcal{D},<}(\mathcal{S}) = \{\delta \in \mathcal{D} : \mathcal{W} \cup \mathit{Conc}(\mathcal{S}) \vdash \mathit{Prem}(\delta)\}$$

3. Example: $\langle \mathcal{W}, \mathcal{D}, < \rangle$ with

   $$\begin{array}{lcl} \mathcal{W} & = & \{B\} \\ \mathcal{D} & = & \{\delta_1, \delta_2\} \\ \delta_1 & = & B \to F \\ \delta_2 & = & P \to \neg F \\ \delta_1 < \delta_2 & & \end{array}$$

   Then

   $$\mathit{Triggered}_{\mathcal{W},\mathcal{D},<}(\emptyset) \quad = \quad \{\delta_1\}$$



5. Conflicted defaults:

$$Conflicted_{\mathcal{W},\mathcal{D},<}(\mathcal{S}) = \{\delta \in \mathcal{D} : \mathcal{W} \cup Conc(\mathcal{S}) \vdash \neg Conc(\delta)\}$$

6. Example (Nixon Diamond):

   Take $\langle \mathcal{W}, \mathcal{D}, < \rangle$ with

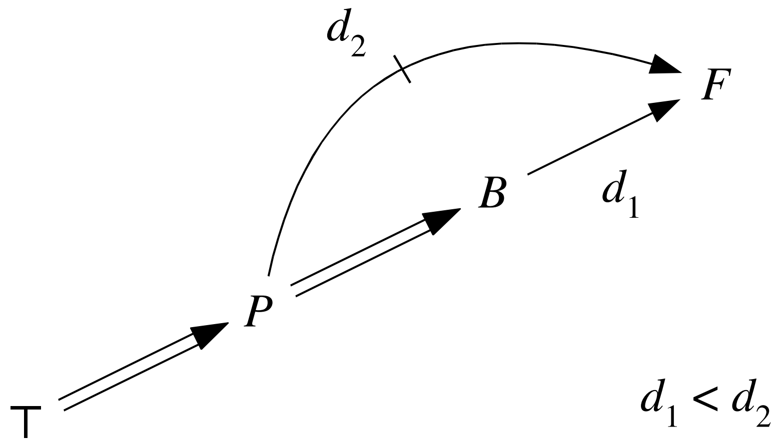
$$\begin{array}{lcl} \mathcal{W} & = & \{Q, R\} \\ \mathcal{D} & = & \{\delta_1, \delta_2\} \\ \delta_1 & = & Q \to P \\ \delta_2 & = & R \to \neg P \\ < & = & \emptyset. \end{array}$$

   Then

$$\begin{array}{lcl} Triggered_{\mathcal{W},\mathcal{D},<}(\emptyset) & = & \{\delta_1, \delta_2\} \\ Conflicted_{\mathcal{W},\mathcal{D},<}(\emptyset) & = & \emptyset \end{array}$$

   But

$$\begin{array}{lcl} Conflicted_{\mathcal{W},\mathcal{D},<}(\{\delta_1\}) & = & \{\delta_2\} \\ Conflicted_{\mathcal{W},\mathcal{D},<}(\{\delta_2\}) & = & \{\delta_1\} \end{array}$$



7. *Basic idea:* A default is defeated if there is a stronger reason supporting a contrary conclusion

$$\mathit{Defeated}_{\mathcal{W},\mathcal{D},<}(\mathcal{S}) = \{\delta \in \mathcal{D} : \exists\, \delta' \in \mathit{Triggered}_{\mathcal{W},\mathcal{D},<}(\mathcal{S}). \\ (1)\ \delta < \delta' \\ (2)\ \mathit{Conc}(\delta') \vdash \neg \mathit{Conc}(\delta)\}.$$

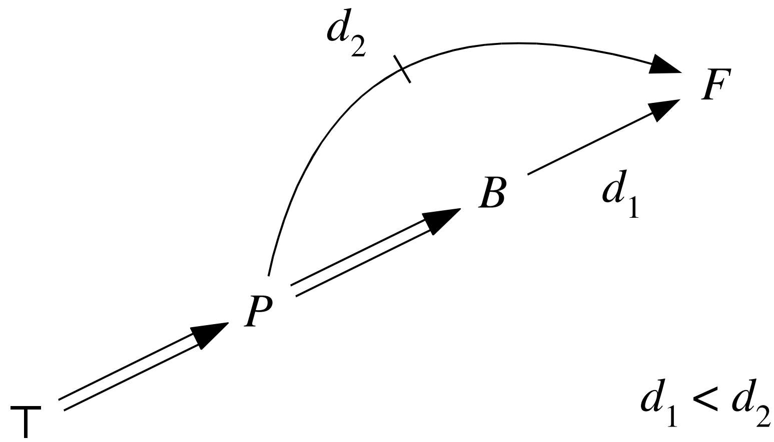
8. Example of defeat (Tweety, again):

   $\langle \mathcal{W}, \mathcal{D}, < \rangle$ where

$$\begin{array}{lcl} \mathcal{W} & = & \{P, P \Rightarrow B\} \\ \mathcal{D} & = & \{\delta_1, \delta_2\} \\ \delta_1 & = & B \to F \\ \delta_2 & = & P \to \neg F \\ \delta_1 < \delta_2 & & \end{array}$$

   Here, δ_1 is *defeated*:

$$\mathit{Defeated}_{\mathcal{W},\mathcal{D},<}(\emptyset) \quad = \quad \{\delta_1\}$$



9. Finally, binding defaults:

$$\begin{aligned} \mathit{Binding}_{\mathcal{W},\mathcal{D},<}(\mathcal{S}) \quad = \quad \{\delta \in \mathcal{D} : \quad & \delta \in \mathit{Triggered}_{\mathcal{W},\mathcal{D},<}(\mathcal{S}) \\ & \delta \notin \mathit{Conflicted}_{\mathcal{W},\mathcal{D},<}(\mathcal{S}) \\ & \delta \notin \mathit{Defeated}_{\mathcal{W},\mathcal{D},<}(\mathcal{S})\} \end{aligned}$$

10. *Stable* scenarios: $\mathcal{S}$ is stable just in case

$$\mathcal{S} = \mathit{Binding}_{\mathcal{W},\mathcal{D},<}(\mathcal{S})$$

11. Example (Tweety, yet again): four scenarios

$$\begin{array}{l} \mathcal{S}_1 = \emptyset \\ \mathcal{S}_2 = \{\delta_1\} \\ \mathcal{S}_3 = \{\delta_2\} \\ \mathcal{S}_4 = \{\delta_1, \delta_2\} \end{array}$$

Only $\mathcal{S}_3 = \{\delta_2\}$ is stable, because

$$\mathcal{S}_3 = \mathit{Binding}_{\mathcal{W},\mathcal{D},<}(\mathcal{S}_3)$$

Three complications

1. Complication #1: Can we just identify the proper scenarios with the stable scenarios?

   Almost . . . but not quite

2. Problem is "groundedness"

   Take $\langle \mathcal{W}, \mathcal{D}, < \rangle$ with

$$\begin{array}{lcl} \mathcal{W} & = & \emptyset \\ \mathcal{D} & = & \{\delta_1\} \\ \delta_1 & = & A \rightarrow A \\ < & = & \emptyset. \end{array}$$

   Then $\mathcal{S}_1 = \{\delta_1\}$ is a stable scenario, but shouldn't be proper

   The belief set generated by $\mathcal{S}_1$ is

$$Th(\mathcal{W} \cup Conc(\mathcal{S})) = Th(\{A\})$$

   but that's not right!

3. Solution:

Let

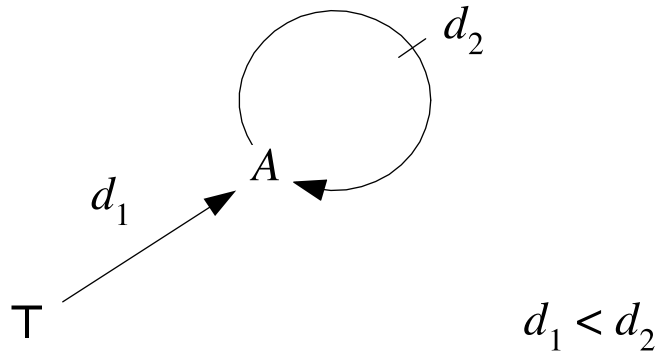
$$Th_{\mathcal{S}}(\mathcal{W}) = \text{Formulas provable from } \mathcal{W} \text{ when ordinary inference rules supplemented with defaults from } \mathcal{S}$$

Then given theory $\langle \mathcal{W}, \mathcal{D}, < \rangle$, define scenario $\mathcal{S}$ as *grounded* in $\mathcal{W}$ iff

$$Th(\mathcal{W} \cup Conc(\mathcal{S})) \subseteq Th_{\mathcal{S}}(\mathcal{W})$$

Finally, given $\langle \mathcal{W}, \mathcal{D}, < \rangle$, define $\mathcal{S}$ as *proper* scenario based on this theory iff

$\mathcal{S}$ is (i) stable and (ii) grounded in $\mathcal{W}$



4. Complication #2: Some theories have *no* proper scenarios, and so no extensions

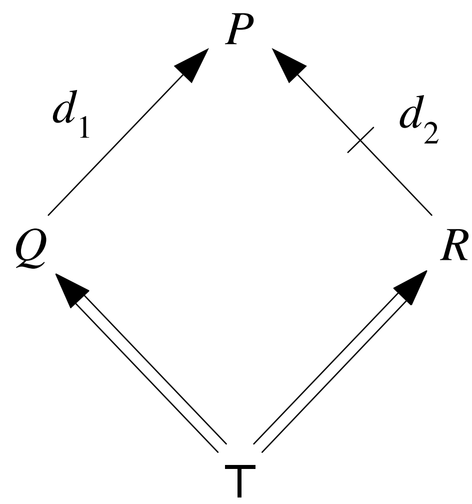
   Example: $\langle \mathcal{W}, \mathcal{D}, < \rangle$ with

$$\begin{array}{lcl} \mathcal{W} & = & \emptyset \\ \mathcal{D} & = & \{\delta_1, \delta_2\} \\ \delta_1 & = & \top \rightarrow A \\ \delta_2 & = & A \rightarrow \neg A \\ \delta_1 < \delta_2 & & \end{array}$$

5. Options:

   Syntactic restrictions to rule out "vicious cycles"

   Move to argumentation framework and opt for "preferred" extensions



6. Complication #3: Some theories have *multiple* proper scenarios, and so multiple extensions

   Example (Nixon Diamond, again):

   Take $\langle \mathcal{W}, \mathcal{D}, < \rangle$ with

$$\begin{array}{lcl} \mathcal{W} & = & \{Q, R\} \\ \mathcal{D} & = & \{\delta_1, \delta_2\} \\ \delta_1 & = & Q \rightarrow P \\ \delta_2 & = & R \rightarrow \neg P \\ < & = & \emptyset. \end{array}$$

   Then *two* proper scenarios

$$\mathcal{S}_1 = \{\delta_1\}$$
$$\mathcal{S}_2 = \{\delta_2\}$$

   and so two extensions:

$$\mathcal{E}_1 = Th(\{Q, R, P\})$$
$$\mathcal{E}_2 = Th(\{Q, R, \neg P\})$$

7. Consider three options:

   A. Choice: pick an arbitrary proper scenario

      Sensible, actually

      But hard to codify as a consequence relation

   B. Brave/credulous: give some weight to any conclusion A contained in *some* extension

      - Endorse A whenever A is contained in some extension

        Example: P and $\neg P$ in Nixon case

      - Endorse $\mathcal{B}(A)$—A is "believable"—whenever A is contained in some extension

        Example: $\mathcal{B}(P)$ and $\mathcal{B}(\neg P)$ in Nixon case

   C. Cautious/"Skeptical": endorse A as conclusion whenever A contained in *every* extension

      Defines reasonable consequence relation: supports neither P nor $\neg P$ in Nixon case

      Note: most popular option, but some problems . . .

8. Complication #3 is *not* a problem for normative interpretation

   Given a default theory Δ, two option for natural deontic logic:

   > Conflict account: Accept $\bigcirc A$ iff $A \in \mathcal{E}$ for some extension $\mathcal{E}$ of Δ
   >
   > (This generalizes van Fraassen)
   >
   > Disjunctive account: Accept $\bigcirc A$ iff $A \in \mathcal{E}$ for some extension $\mathcal{E}$ of Δ
   >
   > (This generalizes Kratzer)

9. Example (Dinner with twins):

Take $\langle \mathcal{W}, \mathcal{D}, < \rangle$ with

$$\begin{array}{lcl} \mathcal{W} & = & \{A1, A2, \neg(D1 \wedge D2)\} \\ \mathcal{D} & = & \{\delta_1, \delta_2\} \\ \delta_1 & = & A1 \rightarrow D1 \\ \delta_2 & = & A2 \rightarrow D2 \\ < & = & \emptyset. \end{array}$$

Two proper scenarios

$$\mathcal{S}_1 = \{\delta_1\}$$

$$\mathcal{S}_2 = \{\delta_2\}$$

and so two extensions:

$$\mathcal{E}_1 = Th(\{A1, A2, \neg(D1 \wedge D2)\}, D1)$$

$$\mathcal{E}_2 = Th(\{A1, A2, \neg(D1 \wedge D2)\}, D2)$$

The upshot is

Conflict account: $\bigcirc(D1)$, $\bigcirc(D2)$

Disjunctive account: $\bigcirc(D1 \vee D2)$

Elaborating default logic

1. Discuss here only two things:

   Ability to reason about priorities

   Treatment of "undercutting" or "exclusionary" defeat

2. Begin with first problem

   So far, fixed priorities on default rules

   But we can reason about default priorities . . . and then use the priorities we arrive at to control our reasoning

3. Five steps:

#1. Add priority statements ($\delta_7 < \delta_9$) to object language

#2. Introduce new *variable priority* default theories

$$\langle \mathcal{W}, \mathcal{D} \rangle$$

with priority statements now belonging to $\mathcal{W}$ and $\mathcal{D}$

#3. Add strict priority axioms to $\mathcal{W}$:

$$\delta < \delta' \Rightarrow \neg(\delta' < \delta)$$
$$(\delta < \delta' \wedge \delta' < \delta'') \Rightarrow \delta < \delta''$$

#4. Lift priorities from object to meta language

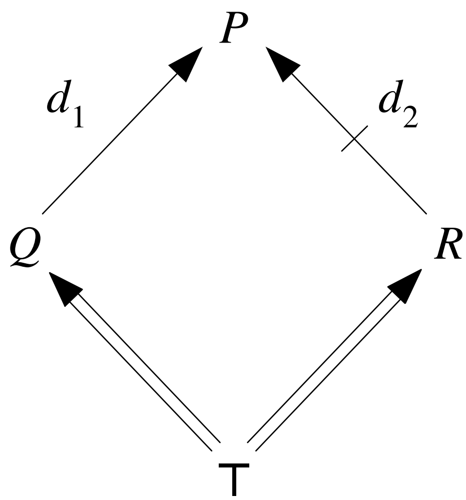
$$\delta <_{\mathcal{S}} \delta' \quad \text{iff} \quad \mathcal{W} \cup Conc(\mathcal{S}) \vdash \delta < \delta'.$$

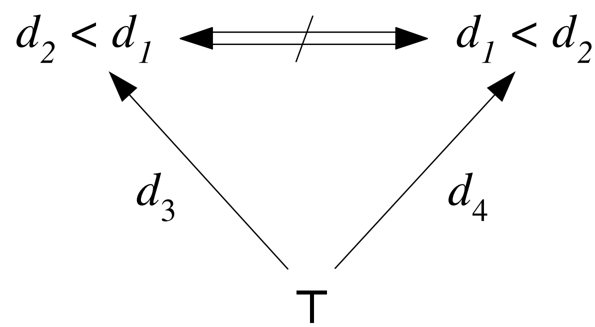
#5. Proper scenarios for new default theories:

$\mathcal{S}$ is a *proper scenario* based on $\langle \mathcal{W}, \mathcal{D} \rangle$

iff

$\mathcal{S}$ is a proper scenario based on $\langle \mathcal{W}, \mathcal{D}, <_{\mathcal{S}} \rangle$





4. Example (Extended Nixon Diamond):

   Consider $\langle \mathcal{W}, \mathcal{D} \rangle$ where

   $\mathcal{W}$ contains Q, P
   $\mathcal{D}$ contains

$$
\begin{aligned}
\delta_1 &= Q \rightarrow P \\
\delta_2 &= R \rightarrow \neg P \\
\delta_3 &= \top \rightarrow \delta_2 < \delta_1 \\
\delta_4 &= \top \rightarrow \delta_1 < \delta_2 \\
\delta_5 &= \top \rightarrow \delta_4 < \delta_3
\end{aligned}
$$

   Then unique proper scenario is

$$\mathcal{S} = \{\delta_1, \delta_3, \delta_5\}$$

5. Example (Perfected security interest):

Consider $\langle \mathcal{W}, \mathcal{D} \rangle$ where

$\mathcal{W}$ contains

$$Possession$$

$$\neg Documents$$

$$Later(\delta_{SMA}, \delta_{UCC})$$

$$Federal(\delta_{SMA})$$

$$State(\delta_{UCC})$$

$\mathcal{D}$ contains

$$\begin{aligned}
\delta_{UCC} &= Possession \rightarrow Perfected \\
\delta_{SMA} &= \neg Documents \rightarrow \neg Perfected \\
\delta_{LP} &= Later(\delta, \delta') \rightarrow \ \delta < \delta' \\
\delta_{LS} &= Federal(\delta) \wedge State(\delta') \rightarrow \ \delta' < \delta \\
\delta_{LSLP} &= \top \rightarrow \ \delta_{LS} < \delta_{LP}
\end{aligned}$$

Unique proper scenario is

$$\mathcal{S} = \{\delta_{LSLP}, \delta_{LP}, \delta_{UCC}\}$$

6. *Undercutting* defeat (epistemology), compared to rebutting defeat

   Example:

   > The object looks red
   >
   > My reliable friend says it is not red
   >
   > Drug 1 makes everything look red

7. *Exclusionary* reasons (practical reasoning)

   Example (Colin's dilemma, from Raz):

   Should son go to private school??

   > The school provides good education
   >
   > He'll meet fancy friends
   >
   > The school is expensive
   >
   > Decision would undermine public education
   >
   > Promise: only consider son's interests ...

8. How can this be represented?

9. Four steps:

#1. New predicate Out, so that $Out(\delta)$ means that δ is undercut, or excluded

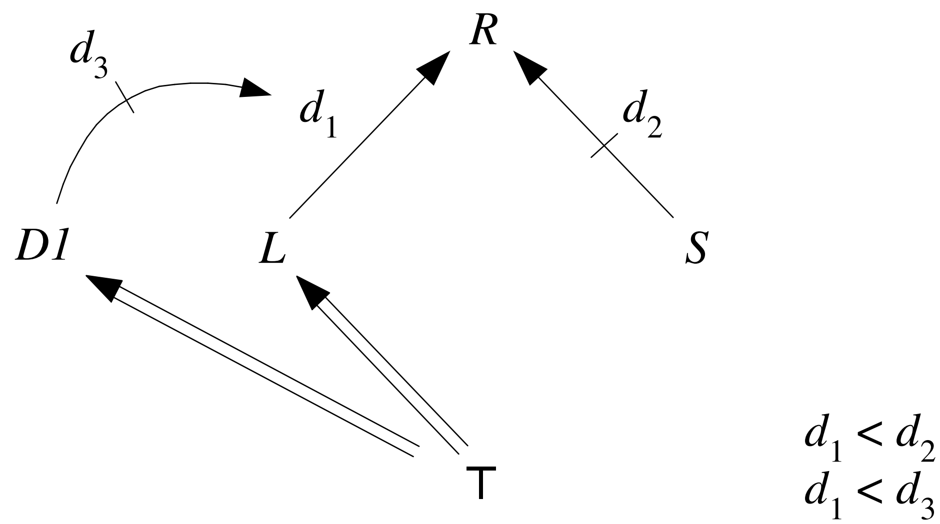
#2. Introduce new *exclusionary* default theories as theories in a language containing Out.

#3. Lift notion of exclusion from object to meta language: where $\mathcal{S}$ is scenario based on theory with $\mathcal{W}$ as hard information

$$\delta \in Excluded_{\mathcal{W},\mathcal{D}.<}(\mathcal{S}) \text{ iff } \mathcal{W} \cup Conc(\mathcal{S}) \vdash Out(\delta)$$

#4. Binding defaults cannot be excluded:

$$\begin{aligned} Binding_{\mathcal{W},\mathcal{D},<}(\mathcal{S}) \quad = \quad \{\delta \in \mathcal{D} : \quad & \delta \in Triggered_{\mathcal{W},\mathcal{D},<}(\mathcal{S}) \\ & \delta \notin Conflicted_{\mathcal{W},\mathcal{D},<}(\mathcal{S}) \\ & \delta \notin Defeated_{\mathcal{W},\mathcal{D},<}(\mathcal{S}) \\ & \delta \notin Excluded_{\mathcal{W},\mathcal{D}.<}(\mathcal{S})\} \end{aligned}$$



11. Example (Drugs):

Take $\langle \mathcal{W}, \mathcal{D} \rangle$ where

$\mathcal{D}$ contains

$$\begin{aligned} \delta_1 &= L \rightarrow R \\ \delta_2 &= S \rightarrow \neg R \\ \delta_3 &= D1 \rightarrow Out(\delta_1) \end{aligned}$$

$\mathcal{W}$ contains L, $D1$, and $\delta_1 < \delta_2$, $\delta_1 < \delta_3$

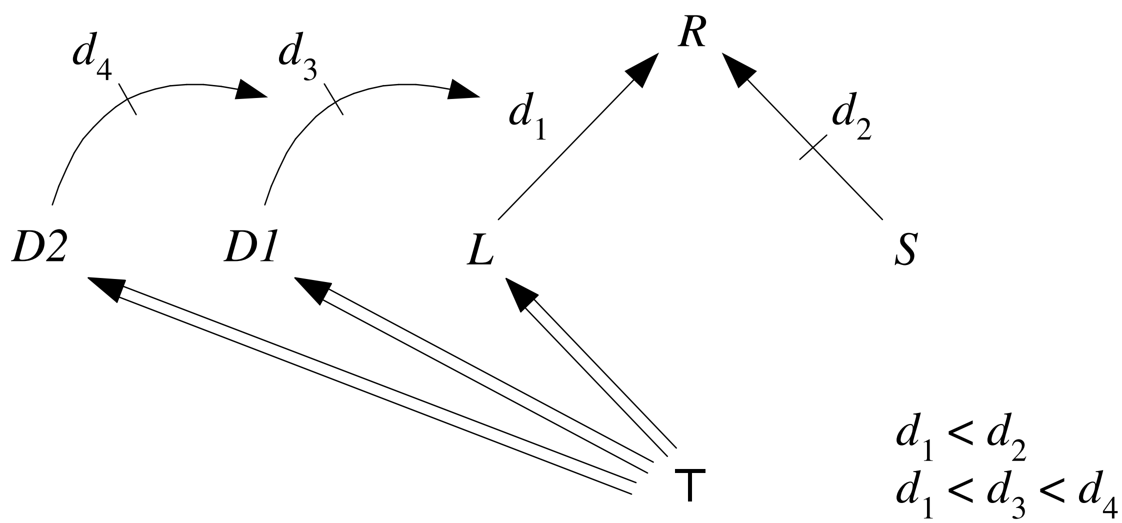
(L = Looks red, R = Red, S = Statement by friend, $D1$ = Drug 1)

So proper scenario is

$$\mathcal{S} = \{\delta_3\}$$

generating the extension

$$\mathcal{E} = Th(\mathcal{W} \cup \{Out(\delta_1)\})$$



12. Example (More drugs):

Take $\langle \mathcal{W}, \mathcal{D} \rangle$ where

$\mathcal{W}$ contains L, $D1$, $D2$, $\delta_1 < \delta_2$, and $\delta_1 < \delta_3 < \delta_4$
$\mathcal{D}$ contains

$$\begin{aligned} \delta_1 &= L \rightarrow R \\ \delta_2 &= S \rightarrow \neg R \\ \delta_3 &= D1 \rightarrow Out(\delta_1) \\ \delta_4 &= D2 \rightarrow Out(\delta_3) \end{aligned}$$

So proper scenario is

$$\mathcal{S} = \{\delta_1, \delta_4\}$$

generating the extension

$$\mathcal{E} = Th(\mathcal{W} \cup \{R, Out(\delta_3)\})$$

13. Example (Colin again):

Let $\mathcal{D}$ contain

$$\begin{aligned}\delta_1 &= E \rightarrow S \\ \delta_2 &= U \rightarrow \neg S \\ \delta_3 &= \neg Welfare(\delta_2) \rightarrow Out(\delta_2)\end{aligned}$$

(E = Provides good education, S = Send son to private school, U = Undermine support for public education)

The default δ_3 is itself an instance of

$$\neg Welfare(\delta) \rightarrow Out(\delta),$$

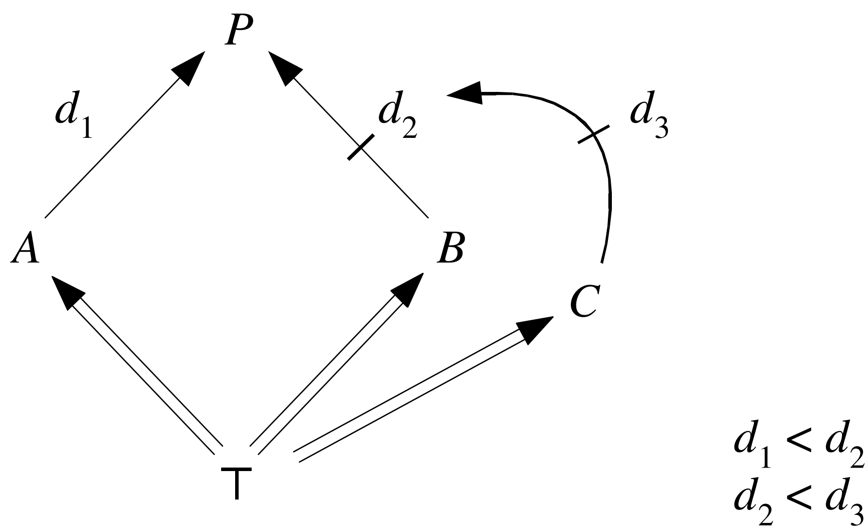
Let $\mathcal{W}$ contain E, U, and $\neg Welfare(\delta_2)$

Then proper scenario is

$$\mathcal{S} = \{\delta_1, \delta_3\}$$

generating the extension

$$\mathcal{E} = Th(\mathcal{W} \cup \{S, Out(\delta_2)\})$$



14. Example (The officers):

$P =$ Some action to perform (or not)

$A =$ Captain's command to perform P

$B =$ Major's command not to perform P

$C =$ Colonel's command to ignore Major's command

Captain $<$ Major $<$ Colonel

Part 3.

Coding Constrainting into Default Logic

Interpreting case bases: fixed priority

1. Factor defaults:

   Where $X \subseteq F^s$ is a factor reason, $X \rightarrow s$ is a factor default

   Example: $\{f_1^\pi, f_2^\pi\} \rightarrow \pi$ is a factor default

2. $\mathcal{F}$ is the set of factor defaults

3. Weak ordering of factor defaults:

   Where r and r' are factor defaults for same side, $r \leq r'$ iff $Prem(r) \subseteq Prem(r')$

   Example: If $r = \{f_1^\pi\} \rightarrow \pi$ and $r' = \{f_1^\pi, f_2^\pi\} \rightarrow \pi$, then $r \leq r'$

4. Preference derived from a case $c = \langle X, r, s \rangle$:

   Where $r' = X^{\overline{s}} \rightarrow \overline{s}$ is strongest rule for losing side, $r'' <_c r'''$ iff $r'' \leq r'$ and $r \leq r'''$.

5. Preference derived from a case base Γ:

   $r <_\Gamma r'$ iff $r <_c r'$ for some $c \in \Gamma$

6. Decision problem: X, Γ

$\Delta_{X,\Gamma} = \langle \mathcal{W}_X, \mathcal{D}_\mathcal{F}, <_\Gamma \rangle$, where

$$\begin{aligned} \mathcal{W}_X &= X \\ \mathcal{D}_\mathcal{F} &= \mathcal{F} \\ <_\Gamma &= \text{preference derived from } \Gamma \end{aligned}$$

7. Fact:

Given X, Γ, and r a factor default favoring side s, then:

> r belongs to a stable scenario based on $\Delta_{X,\Gamma}$ iff $\Gamma \cup \{\langle X, r, s \rangle\}$ is reason consistent

8. Example (from vessel domain):

Recall factors:

f_1^v = subject to Coast Guard regulations
f_2^v = captain and crew
f_3^v = navigation lights
f_4^v = ballast tanks
f_5^v = galley for crew

$f_1^{\overline{v}}$ = no self-propulsion
$f_2^{\overline{v}}$ = primary business not navigation
$f_3^{\overline{v}}$ = not moving at time

Case base $\Gamma = \{c_{11}\}$, where

$c_{11} = \langle X_{11}, r_{11}, s_{11} \rangle$, with

$$X_{11} = \{f_1^v, f_3^v, f_1^{\overline{v}}\}$$
$$r_{11} = \{f_1^{\overline{v}}\} \rightarrow \overline{v}$$
$$s_{11} = \overline{v}$$

so that

$$\{f_1^v, f_3^v\} <_{\Gamma} \{f_1^{\overline{v}}\}$$

New situation:

$$X_{12} = \{f_1^v, f_4^v, f_1^{\overline{v}}\}$$

Relevant factor defaults:

$$\begin{aligned}
r_1 &= \{f_1^v\} \rightarrow v \\
r_2 &= \{f_4^v\} \rightarrow v \\
r_3 &= \{f_1^v, f_4^v\} \rightarrow v \\
r_4 &= \{f_1^{\overline{v}}\} \rightarrow \overline{v}
\end{aligned}$$

So decision problem is

$\Delta_{X_{12},\Gamma} = \langle \mathcal{W}_X, \mathcal{D}_{\mathcal{F}}, <_\Gamma \rangle$, where

$$\begin{aligned}
\mathcal{W}_{X_{12}} &= \{f_1^v, f_4^v, f_1^{\overline{v}}\} \\
\mathcal{D}_{\mathcal{F}} &= \{r_1, r_2, r_3, r_4\} \\
r_1 <_\Gamma r_4 &
\end{aligned}$$

with proper scenarios

$$\begin{aligned}
\mathcal{S}_1 &= \{r_4\} \\
\mathcal{S}_2 &= \{r_2, r_3\}
\end{aligned}$$

Interpreting case bases: variable priorities

1. Value defaults:

   Where r and r' are factor defaults favoring opposite sides, a value default has the form

   $$\top \rightarrow r \prec r'$$

   Example:

   $$r_5 = \top \rightarrow r_4 \prec r_1$$

2. $\mathcal{V}$ is the set of Value defaults

3. Decision problem: $X, \mathcal{V}$

   $\Delta_{X,\mathcal{V}} = \langle \mathcal{W}_X, \mathcal{D}_{\mathcal{F},\mathcal{V}} \rangle$, where

   $$\begin{aligned} \mathcal{W}_X &= X \\ \mathcal{D}_{\mathcal{F},\mathcal{V}} &= \mathcal{F} \cup \mathcal{V} \end{aligned}$$

4. Example:

Recall factors:

f_1^v = subject to Coast Guard regulations
f_3^v = navagation lights
f_4^v = ballast tanks
$f_1^{\overline{v}}$ = no self-propulsion

Recall relevant factor defaults:

$$r_1 = \{f_1^v\} \rightarrow v$$
$$r_2 = \{f_4^v\} \rightarrow v$$
$$r_3 = \{f_1^v, f_4^v\} \rightarrow v$$
$$r_4 = \{f_1^{\overline{v}}\} \rightarrow \overline{v}$$

Recall value default:

$$r_5 = \top \rightarrow r_4 \prec r_1$$

And consider agent with values

$$\mathcal{V}_1 = \{r_5\}$$

confronting the fact situation

$$X_{12} = \{f_1^v, f_4^v, f_1^{\overline{v}}\}$$

So decision problem is

$\Delta_{X_{12},\mathcal{V}_1} = \langle \mathcal{W}_{X_{12}}, \mathcal{D}_{\mathcal{F},\mathcal{V}_1} \rangle$, where

$$\begin{aligned} \mathcal{W}_{X_{12}} &= \{f_1^v, f_4^v, f_1^{\overline{v}}\} \\ \mathcal{D}_{\mathcal{F},\mathcal{V}_1} &= \mathcal{F} \cup \mathcal{V}_1 \\ &= \{r_1, r_2, r_3, r_4\} \cup \{r_5\} \\ &= \{r_1, r_2, r_3, r_4, r_5\} \end{aligned}$$

with proper scenario

$$\mathcal{S}_1 = \{r_1, r_2, r_3, r_5\}$$

5. Case default derived from $c = \langle X, r, s \rangle$:

Where $r' = X^{\overline{s}} \rightarrow \overline{s}$ is strongest rule for losing side, the case default derived from c is

$$c \rightarrow r' \prec r$$

Example: the case default derived from the case

$c_{11} = \langle X_{11}, r_{11}, s_{11} \rangle$, with

$$X_{11} = \{f_1^v, f_3^v, f_1^{\overline{v}}\}$$
$$r_{11} = \{f_1^{\overline{v}}\} \rightarrow \overline{v}$$
$$s_{11} = \overline{v}$$

is

$$r_6 = c_{11} \rightarrow r_7 \prec r_4$$

where

$$r_7 = \{f_1^v, f_3^v\} \rightarrow v$$

6. $\mathcal{C}_\Gamma$ is the set of case defaults derived from Γ

7. Close $\prec$ under $\preceq$ representing weak ordering on defaults

$$(r \prec r' \wedge r' \preceq r'') \supset r \prec r''$$
$$(r \preceq r' \wedge r' \prec r'') \supset r \prec r''$$

Example: Where

$$\begin{aligned} r_1 &= \{f_1^v\} \rightarrow v \\ r_7 &= \{f_1^v, f_3^v\} \rightarrow v \\ r_4 &= \{f_1^{\overline{v}}\} \rightarrow \overline{v} \\ r_6 &= c_{11} \rightarrow r_7 \prec r_4 \end{aligned}$$

Have

$$r_1 \preceq r_7 \text{ and } r_7 \prec r_4$$

So

$$r_1 \prec r_4$$

8. What if value and case defaults conflict?

Example:

$$r_6 = c_{11} \rightarrow r_7 \prec r_4$$

and

$$r_5 = \top \rightarrow r_4 \prec r_1$$

9. Precedent defaults

   Where r is a value default and r' is a case default, a precedent default has the form

$$\top \rightarrow r \prec r'$$

   Example: given value and case defaults

$$\begin{aligned} r_5 &= \top \rightarrow r_4 \prec r_1 \\ r_6 &= c_{11} \rightarrow r_7 \prec r_4 \end{aligned}$$

   then

$$r_8 = \top \rightarrow r_5 \prec r_6$$

   is a precedent default

10. $\mathcal{P}_{\mathcal{V},\Gamma}$ is the entire set of precedent defaults ranking case defaults from $\mathcal{C}_\Gamma$ over value defaults from $\mathcal{V}$

11. Decision problem: $X, \mathcal{V}, \Gamma$

$\Delta_{X,\mathcal{V},\Gamma} = \langle \mathcal{W}_X, \mathcal{D}_{\mathcal{F},\mathcal{V},\Gamma} \rangle$, where

$$\begin{aligned} \mathcal{W}_X &= X \cup \Gamma \\ \mathcal{D}_{\mathcal{F},\mathcal{V},\Gamma} &= \mathcal{F} \cup \mathcal{V} \cup \mathcal{C}_\Gamma \cup \mathcal{P}_{\mathcal{V},\Gamma} \end{aligned}$$

12. Fact:

Given $X, \mathcal{V}, \Gamma$, and r a factor default favoring s, then:

> r belongs to a stable scenario based on $\Delta_{X,\mathcal{V},\Gamma}$ iff $X \cup \{\langle X, r, s \rangle\}$ is reason consistent

13. Example:

Recall factors:

f_1^v = subject to Coast Guard regulations
f_3^v = navagation lights
f_4^v = ballast tanks
$f_1^{\overline{v}}$ = no self-propulsion

Recall relevant factor defaults ($\mathcal{F}$):

$$
\begin{aligned}
r_1 &= \{f_1^v\} \rightarrow v \\
r_2 &= \{f_4^v\} \rightarrow v \\
r_3 &= \{f_1^v, f_4^v\} \rightarrow v \\
r_7 &= \{f_1^v, f_3^v\} \rightarrow v \\
r_4 &= \{f_1^{\overline{v}}\} \rightarrow \overline{v}
\end{aligned}
$$

Recall value default ($\mathcal{V}_1$):

$$r_5 = \top \rightarrow r_4 \prec r_1$$

Add case defaults derived from $\Gamma_1 = \{c_{11}\}$ ($\mathcal{C}_{\Gamma_1}$)

$$r_6 = c_{11} \rightarrow r_7 \prec r_4$$

(and so $r_1 \prec r_4$)

Add precedent defaults comparing case and value defaults ($\mathcal{P}_{\mathcal{V}_1,\Gamma_1}$)

$$r_8 = \top \rightarrow r_5 \prec r_6$$

And consider again the situation

$$X_{12} = \{f_1^v, f_4^v, f_1^{\overline{v}}\}$$

So decision problem is

$\Delta_{X_{12},\mathcal{V}_1,\Gamma_1} = \langle \mathcal{W}_{X_{12}}, \mathcal{D}_{\mathcal{F},\mathcal{V}_1,\Gamma_1} \rangle$, where

$$\begin{aligned}
\mathcal{W}_{X_{12}} &= X_{12} \cup \Gamma_1 \\
&= \{f_1^v, f_4^v, f_1^{\overline{v}}\} \cup \{c_{11}\} \\
&= \{f_1^v, f_4^v, f_1^{\overline{v}}, c_{11}\}
\end{aligned}$$

$$\begin{aligned}
\mathcal{D}_{\mathcal{F},\mathcal{V}_1,\Gamma_1} &= \mathcal{F} \cup \mathcal{V}_1 \cup \mathcal{C}_{\Gamma_1} \cup \mathcal{P}_{\mathcal{V}_1,\Gamma_1} \\
&= \{r_1, r_2, r_3, r_4\} \cup \{r_5\} \cup \{r_6\} \cup \{r_8\} \\
&= \{r_1, r_2, r_3, r_4, r_5, r_6, r_8\}
\end{aligned}$$

with proper scenarios

$$\mathcal{S}_1 = \{r_4, r_6, r_8\}$$
$$\mathcal{S}_2 = \{r_2, r_3, r_6, r_8\}$$

Stepping back a bit . . .

1. Suppose court now makes decision for v ("vessel") on the basis of r_2:

   "It's a vessel because it has ballast tanks"

   This has a dynamic effect, updating case base to

   $$\Gamma_2 = \Gamma_1 \cup \{\langle X_{12}, r_2, v\rangle\}$$

   and so changing the meaning of "vessel" so that, in future cases, $\{f_1^{\bar{v}}\}$ can never outweigh $\{f_4^v\}$ . . . nor can it outweigh $\{f_1^v, f_4^v\}$, $\{f_3^v, f_4^v\}$, $\{f_1^v, f_3^v, f_4^v\}$, etc

2. Making the same decision on the basis of r_3

   "It's a vessel because it is subject to Coast Guard regulations and has ballast tanks"

   has as similar effect, but less extensive

3. Even applying the existing rule r_4

   "It's not a vessel because it's not self-propelled"

   changes the meaning of the term, by changing underlying constraints

4. Conjecture:

This sort of constraint reflects a principle of *conversational integrity* at work whenever we say, as part of a conversation, things like

John is bald
Sarah is a good student
That apple is large
Target shooting is a sport

but simply placed under a microscope in discussion of legal precedent

5. Upshot:

The "justification of precedent" might have to do not so much with concerns of

equity
predictibility
efficiency

as we the application of this general conversational principle: we want to make sure the courts are having the same conversation