Class Notes, PHIL 478: Logics for Defeasible Reasoning

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Contents

1 Default logic

1.1 Default rules

- Background language, logical closure
- Rules of form $X \to Y$
- Where $\delta = X \to Y$, have $Premise(\delta) = X$, $Conclusion(\delta) = Y$. Also, if $\mathcal{S}$ set of defaults, have $Conclusion(\mathcal{S}) = \{Conclusion(\delta) : \delta \in \mathcal{S}\}$
- Priorities

1.2 Fixed priority default theories

- **Definition 1 (Fixed priority default theories)** A fixed priority default theory Δ is a structure of the form $\langle \mathcal{W}, \mathcal{D}, < \rangle$, in which $\mathcal{W}$ is a set of ordinary formulas, $\mathcal{D}$ is a set of default rules, and $<$ is a strict partial ordering on $\mathcal{D}$.
- **Definition 2 (Extensions)** Let $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ be a fixed priority default theory. Then $\mathcal{E}$ is an extension of Δ just in case, for some proper scenario $\mathcal{S}$ based on this theory,

$$\mathcal{E} = Th(\mathcal{W} \cup Conclusion(\mathcal{S})).$$

1.3 Stability

- **Definition 3 (Triggered defaults)** Let $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ be a fixed priority default theory, and $\mathcal{S}$ a scenario based on this theory. Then the defaults from $\mathcal{D}$ that are *triggered* in the context of the scenario $\mathcal{S}$ are those belonging to the set

$$Triggered_{\mathcal{W},\mathcal{D}}(\mathcal{S}) = \{\delta \in \mathcal{D} : \mathcal{W} \cup Conclusion(\mathcal{S}) \vdash Premise(\delta)\}.$$

- **Definition 4 (Conflicted defaults)** Let $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ be a fixed priority default theory, and $\mathcal{S}$ a scenario based on this theory. Then the defaults from $\mathcal{D}$ that are conflicted in the context of the scenario $\mathcal{S}$ are those belonging to the set

$$Conflicted_{\mathcal{W},\mathcal{D}}(\mathcal{S}) = \{\delta \in \mathcal{D} : \mathcal{W} \cup Conclusion(\mathcal{S}) \vdash \neg\, Conclusion(\delta)\}.$$

- **Definition 5 (Defeated defaults: preliminary definition)** Let $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ be a fixed priority default theory, and $\mathcal{S}$ a scenario based on this theory. Then the defaults from $\mathcal{D}$ that are defeated in the context of the scenario $\mathcal{S}$ are those belonging to the set

$$\begin{aligned} \mathit{Defeated}_{\mathcal{W},\mathcal{D},<}(\mathcal{S}) \quad = \quad \{\delta \in \mathcal{D} : \ & \text{there is a default } \delta' \in \mathit{Triggered}_{\mathcal{W},\mathcal{D}}(\mathcal{S}) \text{ such that} \\ & (1)\ \delta < \delta', \\ & (2)\ \mathcal{W} \cup \{\mathit{Conclusion}(\delta')\} \vdash \neg\, \mathit{Conclusion}(\delta)\}. \end{aligned}$$

- **Definition 6 (Binding defaults)** Let $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ be a fixed priority default theory, and $\mathcal{S}$ a scenario based on this theory. Then the defaults from $\mathcal{D}$ that are binding in the context of the scenario $\mathcal{S}$ are those belonging to the set

$$\begin{aligned} \mathit{Binding}_{\mathcal{W},\mathcal{D},<}(\mathcal{S}) \quad = \quad \{\delta \in \mathcal{D} : \quad & \delta \in \mathit{Triggered}_{\mathcal{W},\mathcal{D}}(\mathcal{S}), \\ & \delta \notin \mathit{Conflicted}_{\mathcal{W},\mathcal{D}}(\mathcal{S}), \\ & \delta \notin \mathit{Defeated}_{\mathcal{W},\mathcal{D},<}(\mathcal{S})\}. \end{aligned}$$

- **Definition 7 (Stable scenarios)** Let $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ be a fixed priority default theory, and $\mathcal{S}$ a scenario based on this theory. Then $\mathcal{S}$ is a stable scenario based on the theory Δ just in case

$$\mathcal{S} = \mathit{Binding}_{\mathcal{W},\mathcal{D},<}(\mathcal{S}).$$

1.4 Proper scenarios and extensions

- **Definition 8 (Approximating sequences)** Let $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ be a fixed priority default theory and $\mathcal{S}$ a scenario based on this theory. Then $\mathcal{S}_0, \mathcal{S}_1, \mathcal{S}_2, \ldots$ is an approximating sequence that is based on the theory Δ and constrained by the scenario $\mathcal{S}$ just in case

$$\begin{aligned} \mathcal{S}_0 \quad &= \quad \emptyset, \\ \mathcal{S}_{i+1} \quad &= \quad \{\delta : \ \delta \in \mathit{Triggered}_{\mathcal{W},\mathcal{D}}(\mathcal{S}_i), \\ & \qquad\quad \delta \notin \mathit{Conflicted}_{\mathcal{W},\mathcal{D}}(\mathcal{S}), \\ & \qquad\quad \delta \notin \mathit{Defeated}_{\mathcal{W},\mathcal{D},<}(\mathcal{S})\}. \end{aligned}$$

- **Definition 9 (Proper scenarios)** Let Δ be a default theory and $\mathcal{S}$ a scenario based on this theory, and let $\mathcal{S}_0, \mathcal{S}_1, \mathcal{S}_2, \ldots$ be an approximating sequence that is based on Δ and constrained by $\mathcal{S}$. Then $\mathcal{S}$ is a proper scenario based on Δ just in case $\mathcal{S} = \bigcup_{i \geq 0} \mathcal{S}_i$.

- **Theorem 1** Let $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ be a fixed priority default theory and $\mathcal{S}$ a proper scenario based on this theory. Then $\mathcal{S}$ is also a stable scenario based on the theory Δ.

- **Theorem 2** A fixed priority default theory $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ has an inconsistent extension just in case $\mathcal{W}$ is inconsistent.

- **Theorem 3** If a fixed priority default theory has an inconsistent extension, this is its only extension.

- **Theorem 4** Let $\mathcal{S}$ and $\mathcal{R}$ be proper scenarios based on a fixed priority default theory, with $\mathcal{R} \subseteq \mathcal{S}$. Then $\mathcal{R} = \mathcal{S}$.

- **Theorem 5** Let $\mathcal{E}$ be an extension of the fixed point default theory $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$, and suppose $\mathcal{A} \subseteq \mathcal{W}$. Then $\mathcal{E}$ is is also an extension of the theory $\Delta' = \langle \mathcal{W} \cup \mathcal{A}, \mathcal{D}, < \rangle$.

1.5 Some consequence relations

- **Definition 10 (Credulous consequence)** Let Δ be a default theory. Then Y is a credulous consequence of Δ—written, $\Delta \mathrel{|\!\sim}_C Y$—just in case $Y \in \mathcal{E}$ for some extension $\mathcal{E}$ of Δ.

- **Definition 11 (Skeptical consequence)** Let Δ be a default theory. Then Y is a skeptical consequence of Δ—written, $\Delta \mathrel{|\!\sim}_S Y$—just in case $Y \in \mathcal{E}$ for each extension $\mathcal{E}$ of Δ.

- Note that credulous consequence is crazy, in the epistemic case.

- **Observation 1**

  If $\langle \mathcal{W}, \mathcal{D}, < \rangle \mathrel{|\!\sim}_S A$ and $\langle \mathcal{W} \cup \{A\}, \mathcal{D}, < \rangle \mathrel{|\!\sim}_S B$, then $\langle \mathcal{W}, \mathcal{D}, < \rangle \mathrel{|\!\sim}_S B$.

1.6 Defeasible arguments

- **Definition 12 (Defeasible arguments)** Where $\mathcal{S}$ is a set of default rules and $\mathcal{W}$ is a set of propositions, a defeasible argument, originating from $\mathcal{W}$ and constructed from $\mathcal{S}$, is a sequence of propositions $X_1, X_2, \ldots, X_n$ such that each member X_i of the sequence satisfies one of the following conditions: (1) X_i is an axiom of propositional logic; (2) X_i belongs to $\mathcal{W}$; (3) X_i follows from previous members of the sequence by modus ponens; or (4) there is some default δ from $\mathcal{S}$ such that $Conclusion(\delta)$ is X_i and $Premise(\delta)$ is a previous member of the sequence.

- **Definition 13 (Argument extensions)** Let $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ be a fixed priority default theory. Then Φ is an argument extension of Δ just in case, for some proper scenario $\mathcal{S}$ based on this theory,

$$\Phi = Argument_{\mathcal{W}}(\mathcal{S}).$$

- **Definition 14 (Grounded scenarios)** Let $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ be a fixed priority default theory and $\mathcal{S}$ a scenario based on this theory. Then $\mathcal{S}$ is *grounded* in the theory Δ just in case $Th(\mathcal{W} \cup Conclusion(\mathcal{S})) \subseteq Conclusion(Argument_{\mathcal{W}}(\mathcal{S}))$.

- **Theorem 6** Let $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ be a fixed priority default theory and $\mathcal{S}$ a proper scenario based on this theory. Then $\mathcal{S}$ is also grounded in the theory Δ.

- **Theorem 7** Let $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ be a fixed priority default theory. Then $\mathcal{S}$ is a proper scenario based on the theory Δ just in case $\mathcal{S}$ is both stable and also grounded in this theory.

1.7 Reiter default theories

- A *Reiter default* is a rule of the form $(A : C \,/\, B)$.

- If δ is the Reiter default above, then $Premise(\delta) = A$, $Conclusion(\delta) = B$, $Justification(\delta) = C$.

- **Definition 15 (Reiter default theories)** A Reiter default theory Δ is a structure of the form $\langle \mathcal{W}, \mathcal{D} \rangle$, in which $\mathcal{W}$ is a set of ordinary formulas and $\mathcal{D}$ is a set of Reiter default rules.

- **Definition 16 (R-conflicted defaults)** Let $\Delta = \langle \mathcal{W}, \mathcal{D} \rangle$ be a Reiter default theory, and $\mathcal{S}$ a scenario based on this theory. Then the defaults from $\mathcal{D}$ that are R-conflicted in the context of the scenario $\mathcal{S}$ are those belonging to the set

$$R\text{-}conflicted_{\mathcal{W},\mathcal{D}}(\mathcal{S}) = \{\delta \in \mathcal{D} : \mathcal{W} \cup Conclusion(\mathcal{S}) \vdash \neg\, Justification(\delta)\}.$$

- **Definition 17 (Approximating sequences)** Let $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ be a Reiter default theory and $\mathcal{S}$ a scenario based on this theory. Then $\mathcal{S}_0, \mathcal{S}_1, \mathcal{S}_2, \ldots$ is an approximating sequence that is based on the theory Δ and constrained by the scenario $\mathcal{S}$ just in case

$$\begin{aligned} \mathcal{S}_0 &= \emptyset, \\ \mathcal{S}_{i+1} &= \{\delta : \ \delta \in Triggered_{\mathcal{W},\mathcal{D}}(\mathcal{S}_i), \\ &\qquad\qquad \delta \notin R\text{-}conflicted_{\mathcal{W},\mathcal{D}}(\mathcal{S}), \end{aligned}$$

- Not all Reiter default theories have proper scenarios, and so not all have extensions.

1.8 Normal default theories

- A normal default is a default of the form $A \rightarrow B$.

- A normal default can also be identified with a Reiter default of the form $(A : B \,/\, B)$. If the default δ is normal, then $Justification(\delta) = Conclusion(\delta)$.

- **Definition 18 (Normal default theories)** A normal default theory can be defined as either (A) a prioritized default theory whose priority ordering is empty, or as (B) a Reiter default theory containing only normal defaults.

- **Theorem 8** Every normal default theory has an extension.