

Sets, relations, functions

1. Some definitions and facts:

$$A \subseteq B \text{ =df } \forall x (x \in A \rightarrow x \in B)$$

$$A = B \text{ =df } A \subseteq B \wedge B \subseteq A$$

$$A \subset B \text{ =df } A \subseteq B \wedge \sim (B \subseteq A)$$

$$A \cup B \text{ =df } \{x : x \in A \vee x \in B\}$$

$$A \cap B \text{ =df } \{x : x \in A \wedge x \in B\}$$

$$A - B \text{ =df } \{x : x \in A \wedge x \notin B\}$$

$$\emptyset \text{ governed by axiom: } \sim \exists x (x \in \emptyset)$$

$$U \text{ governed by axiom: } \forall x (x \in U)$$

$$\mathcal{P}(A) \text{ =df } \{B : B \subseteq A\}$$

$$\{a_1, a_2, \dots, a_n\} \text{ =df } \{x : x = a_1 \vee x = a_2 \vee \dots \vee x = a_n\}$$

2. Naive abstraction (and un-abstraction):

$$\frac{A^a/x}{a \in \{x: A\}} \in I$$

$$\frac{a \in \{x: A\}}{A^a/x} \in E.$$

3. Cartesian products, relations:

$$X \times Y = \{ \langle a, b \rangle : a \in X \wedge b \in Y \}$$

$$X_1 \times X_2 \times \dots \times X_n = \{ \langle a_1, \dots, a_n \rangle : a_i \in X_i \}$$

A relation R is a subset of an n -ary Cartesian product.

Eg: $R \subseteq X \times Y$. Here, X is the domain and Y the range of the relation.

4. Functions

A function $f: X \rightarrow Y$ is a relation $f \subseteq X \times Y$ subject to condition that

$$\forall a, b, c \left[(\langle a, b \rangle \in f \wedge \langle a, c \rangle \in f) \rightarrow b = c \right].$$

We say that: $f(a) = b$