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ORIGINAL RESEARCH

Reasoning with dimensions and magnitudes

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Abstract

This paper shows how two models of precedential constraint can be broadened to include legal information represented through dimensions. I begin by describing a standard representation of legal cases based on boolean factors alone, and then reviewing two models of constraint developed within this standard setting. The first is the "result model", supporting only a fortiori reasoning. The second is the "reason model", supporting a richer notion of constraint, since it allows the reasons behind a court's decisions to be taken into account. I then show how the initial representation can be modified to incorporate dimensional information and how the result and reason models can be adapted to this new dimensional setting. As it turns out, these two models of constraint, which are distinct in the standard setting, coincide once they are transposed to the new dimensional setting, yielding exactly the same patterns of constraint. I therefore explore two ways of refining the reason model of constraint so that, even in the dimensional setting, it can still be separated from the result model.

Keywords Precedent · Constraint · Dimensions

1 Introduction

One of the most distinctive features of artificial intelligence and law, as a field, is the interplay found there between case-based and rule-based, or logical, methods of reasoning. The application of case-based reasoning in the legal domain is exemplified by the line of research originating in Edwina Rissland and Kevin Ashley's HYPO system, first sketched in Rissland and Ashley (1987), later elaborated and described most completely in Ashley (1990). The goal of that system was to model reasoning on the basis of precedent, by identifying previous cases supporting a position in some situation, constructing arguments on the basis of those cases, and anticipating responses to those arguments. Rule-based legal reasoning has a longer history, with speculative roots going back at least to Layman Allen's (1957), but was first

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shown to be viable in a series of projects—highlights include Sergot et al. (1986) and Bench-Capon et al. (1987)—aimed at representing legislative and regulatory information in logic programs.

These two forms of reasoning, case-based and rule-based, are often seen as competitors, sometimes hostile competitors, in many areas of artificial intelligence, but relations have been more harmonious in the subfield of artificial intelligence and law, where there is a tradition of reconciliation. Tentative efforts in this direction can traced back to Jaap Hage (1993) and Ronald Loui et al. (1993), but substantial progress was first achieved by Henry Prakken and Giovanni Sartor (1998), who showed that many of the patterns of legal reasoning and legal argument first studied in the case-based framework of HYPO—as well as in successor systems, such as CABARET (Rissland and Skalak 1989) and CATO (Aleven and Ashley 1997)—could also be modeled in the framework of a defeasible logic with variable priorities. This unification of ideas from case-based reasoning with ideas from the rule-based framework of defeasible logic is one of the great success stories of the entire field.

But problems remain—or at least barriers to reconciliation—deriving from the differences, both in case-based knowledge representation and in the reasoning it supports, between "factors" and "dimensions". Usage in the field is not entirely uniform, but let us say, for present purposes, that a *factor* is a legally significant proposition, which may or may not hold in a given situation, but which, when it does hold, always favors the same side in a dispute. A *dimension*, by contrast, is an ordered set of legally significant *values*, where the ordering among values reflects the extent to which that value favors one side or the other. While a factor is binary, then, either holding or not in any situation, and uniform in polarity, always favoring a single side when it does hold, a dimension can take on different values in different situations, and the polarity associated with a given value can be unclear.

This contrast between factors and dimensions can be illustrated with examples from the field of trade secrets law, the original application domain of the HYPO and CATO systems, which explores the conditions under which a defendant can be said to have gained an unfair competitive advantage over a plaintiff through the misappropriation of a trade secret. Here, one relevant consideration bearing on the plaintiff's claim that some body of information constitutes a trade secret is whether the defendant has, or has not, signed a non-disclosure agreement. This consideration can naturally be represent as a factor—say, the proposition that a non-disclosure agreement was signed—since this proposition either holds or does not hold, and, if it does, always supports the plaintiff's contention that the information is a genuine trade secret. Another consideration concerns the extent to which the information alleged to be a trade secret has already been disclosed to outsiders. This consideration is best represented as a dimension, with the possible values along that dimension—the number of outsiders to whom the information was disclosed—arranged in such a way that disclosure to more and more outsiders progressively strengthens the case for the defendant, since it provides stronger support for the idea that the information in question was not in fact a secret.

A third consideration concerns measures taken to protect the information purported to be a secret—again best represented as a dimension, with protective measures as values, and these values ordered in such a way that stronger measures provide stronger support for the plaintiff's claim that the information was indeed a secret. Imagine that, in some case, the information in question is data stored on a disk, and consider four possible values along the protective measures dimension: (1) the plaintiff has taken no protective measures, (2) the plaintiff has encrypted the disk, (3) the plaintiff has locked the disk in a safe, (4) the plaintiff has both encrypted the disk and locked it in a safe. These four values could naturally be ordered so that the second and third provide stronger support for the plaintiff than the first, but are incomparable to each other, and the fourth provides stronger support for the plaintiff than all the others.

The last example highlights three useful points about dimensions. It shows, first of all, that the values along a dimension need not correspond to a numerical range, but can be entirely qualitative, and second, that the ordering among these values need not be linear. Third, the example provides a clear illustration of the fact that the polarity of some particular value along a dimension can be indeterminate. Consider a case in which the protective measures dimension takes the second value listed above: the disk was encrypted, but not locked away. It is easy to imagine the plaintiff arguing that this value assignment supports the conclusion that the information was a secret, since, after all, it was encrypted. It is also easy to imaging the defendant arguing that the same value assignment supports the conclusion that the information was not a secret, since it was merely encrypted, and not locked away as well.

Given this distinction between factors and dimensions, then, what problem does it pose for Prakken and Sartor's reconstruction of case-based reasoning in a rule-based setting? Simply this: On one hand, many major case-based systems, with the notable exception of CATO, support reasoning based on dimensions, not just factors, and most researchers in the field, including the authors of CATO, believe that full dimensional resources are necessary for an adequate representation of legal information.[1] But on the other hand, the logical reconstruction of ideas from case-based reasoning offered by Prakken and Sartor takes only factors into account—relying on rules whose premises are conjunctions of factors alone, without analysis of the connection between these factors and the underlying dimensional information.

The point can be illustrated with one of Prakken and Sartor's own hypothetical examples concerning the issue whether an individual who has spent time in another country has changed fiscal domicile with respect to income tax. Among the considerations bearing on this issue is the duration of the individual's stay abroad, where greater duration provides stronger support for change of domicile. Here, it is natural to represent duration through a dimension that can take on a variety of values—the individual might have stayed in the other country for a week, a month, six months, a year, five years, and so on. But in fact, Prakken and Sartor bypass the full range of available values and deal with this dimensional information, instead, only through the introduction of a pair of factors—"long-duration" and "short-duration"—where the first favors change of fiscal domicile and the second favors no change. Now of course, it is sensible to assume that a long duration abroad should favor change of domicile, and that a short duration should favor no change, but that leaves open the

[1] See both Bench-Capon and Rissland (2001) and Rissland and Ashley (2002) for arguments supporting the importance of dimensions in legal knowledge representation.

question whether any particular duration should count as long or as short? Suppose an individual has lived in another country for thirteen months. Does a period abroad of that length count as a long duration, and so favor change of domicile, or does it count as a short duration, and so favor lack of change? This is exactly the kind of question that should itself be subject to legal argument, rather than settled through representational convention.

Even if the problem of relating factors to underlying dimensional information is real, however, it may appear to be only a minor problem, and easily solvable. This seems to be what Prakken and Sartor themselves thought.[2] But in a brief and, I feel, somewhat neglected paper, Trevor Bench-Capon (1999) sets out a number of arguments that raise real concerns about the possibility of handling dimensional information in the kind of rule-based systems used by Prakken and Sartor. As Bench-Capon sees it, there are two crucial problems: first, as I have already emphasized, no particular value along a dimensional scale necessarily favors one side or the other, in the way that factors do, and second, that if several dimensions are present, strength along one dimension can be traded off for strength along another.

The past few years have seen a renewed interest in accounting for dimensional information.[3] As far as I know, however, the problems raised by Bench-Capon concerning the representation of this information in a rule-based setting have not yet been explicitly addressed, let alone resolved. My goal in this paper is to do just that—to propose one way in which dimensional reasoning can be modeled in a rule-based framework. In carrying out this project, I will not be working directly with Prakken and Sartor's logic, which is designed to model legal argument, but instead in a rule-based framework of my own, designed to characterize the concept of legal constraint. But the proposals set out here can be adapted to Prakken and Sartor's logic, or to any other framework in which the reasons underlying common law decisions are carried by defeasible rules.

The paper proceeds as follows: I begin, in the next section, by describing a standard representation of legal cases based on factors alone, and then reviewing two models of precedential constraint developed within this standard setting. The first is the "result model" of constraint, supporting only a fortiori reasoning and considered, by many, as too weak to be plausible.[4] The second is what I call the "reason model," supporting a richer notion of constraint, since it allows the reasons behind a court's decisions to be taken into account. In Sect. 3, I show, first of all, how the initial representation can be modified to incorporate dimensional information rather than standard factors, and then how the result and reason models of constraint can be adapted to this new dimensional setting. As it turns out, the two models of constraint explored here, which are distinct in the standard setting, coincide once they are transposed to the new dimensional setting, yielding exactly the same patterns of constraint. In Sect. 4, therefore, I explore two ways of refining the reason model of

[2] Speaking of dimensions, as well as hypotheticals, they write that "there are no theoretical objections to extending our analysis with these features" (Prakken and Sartor 1998, p. 279).

[3] See, for example, Al-Abdulkarim et al. (2016) and Prakken et al. (2015).

[4] The phrase "result model" is due to Alexander (1989).

constraint so that, even in the dimensional setting, it can still be separated from the result model. The paper has two appendices. The first describes one way of interpreting standard information within the dimensional setting and explores relations, in this setting, between the two models of constraint. The second verifies formal observations found in the text.

2 The standard setting

2.1 Factors, rules, and cases

Let us begin by postulating a set F of *standard factors*—the phrase is meant to distinguish these factors from a different kind of factor to be introduced later on, which will be related to dimensional information. We simplify by imagining that the reasoning under consideration involves only a single step, proceeding at once from the factors present in a case to a decision for the plaintiff or defendant, rather than moving through a series of intermediate legal concepts. As a result, we can suppose that the entire set of standard factors is exhausted by the set $F^{\pi} = \{f_1^{\pi}, \ldots, f_n^{\pi}\}$ of those favoring the plaintiff together with the set $F^{\delta} = \{f_1^{\delta}, \ldots, f_m^{\delta}\}$ of those favoring the defendant: $F = F^{\pi} \cup F^{\delta}$. As this notation suggests, we take π and δ to represent the two sides in a dispute, plaintiff and defendant, and where s is one of these sides, we let $\overline{s}$ represent the other: $\overline{\pi} = \delta$ and $\overline{\delta} = \pi$.

Within the standard setting, a situation confronting the court—that is, a *standard fact situation*—can be defined simply as some particular subset X of the standard factors: $X \subseteq F$. And where X is a standard fact situation, we let X^s represent the standard factors from X that support the side s: $X^{\pi} = X \cap F^{\pi}$ and $X^{\delta} = X \cap F^{\delta}$.

Rules will be defined in terms of reasons, where a *standard reason favoring the side s* is some set of factors favoring that side. To illustrate: $\{f_1^{\pi}, f_2^{\pi}\}$ is a standard reason favoring the plaintiff, while $\{f_1^{\delta}\}$ is a standard reason favoring the defendant; but the set $\{f_1^{\pi}, f_1^{\delta}\}$ is not a reason, since the factors it contains do not uniformly favor one side or another. Reasons of this kind are to be interpreted conjunctively, so that, for example, the reason $\{f_1^{\pi}, f_2^{\pi}\}$ represents the conjunction of the propositions represented by the factors f_1^{π} and f_2^{π}, and the reason $\{f_1^{\pi}\}$ carries the same meaning as the factor f_1^{π}.

The idea that a factor holds in a particular situation, or that the situation satisfies that factor, can be defined very simply in the standard case, and then lifted from factors to reasons, or sets of factors, by stipulating that a situation satisfies a set of factors whenever it satisfies each factor from that set.

Definition 1 (*Factor satisfaction: standard*) Where X is a standard fact situation and f_n^s is a standard factor, X satisfies f_n^s—written, $X \models f_n^s$—if and only if f_n^s belongs to X.

Definition 2 (*Reason satisfaction*) Where X is a fact situation and W is a matching reason, X satisfies W—written, $X \models W$—if and only if X satisfies each factor contained in W.

We can then define a relation of entailment between reasons, by stipulating that one reason entails another whenever any situation that satisfies the first of these reasons also satisfies the second.

Definition 3 (*Reason entailment*) Where W and Z are matching reasons, W entails Z—written, $W \Vdash Z$—if and only if $X \models Z$ whenever $X \models W$, for any matching fact situation X.

Definitions 2 and 3 call for a few comments. First, both contain the requirement that reasons, or reasons and fact situations, must be matching. This requirement can be ignored for now, since we are so far working only in the standard setting, with standard reasons and standard fact situations. In the next section, the same definitions will carry over to the dimensional setting, with dimensional reasons and fact situations. Once dimensional information is introduced, the matching requirement is meant to ensure that these definitions should apply only within a single setting, not across settings. Second, it is easy to see, if X is a standard fact situation and W and Z are standard reasons, that X satisfies W just in case $W \subseteq X$, and that W entails Z just in case $Z \subseteq W$. Why not, then, bypass factor satisfaction and define reason satisfaction and reason entailment directly in terms of these simple set inclusions? The answer is that we are aiming at definitions that will generalize to the dimensional setting, and these simple set inclusion definitions, unlike the current Definitions 2 and 3, would not do so. And finally, the relation of logical entailment set out in Definition 3 corresponds to an intuitive comparison of strength among reasons: where W and Z are reasons supporting the same side, it is natural to suppose that W *is at least as strong a reason as* Z *for that side* just in case $W \Vdash Z$.

Given our notion of a standard reason, a *standard rule* r can be defined as a statement of the form $W \rightarrow s$, where W is a standard reason supporting the side s. We introduce two functions—*Premise* and *Conclusion*—picking out the premise and the conclusion of a rule, so that, in the case of this particular rule r we would have $Premise(r) = W$ and $Conclusion(r) = s$. A rule like this is to be interpreted as defeasible, telling us that its premise entails its conclusion, not as a matter of necessity, but only by default. What the rule $W \rightarrow s$ means is that, if the factors from W hold in some fact situation, then as a default, the court ought to reach a decision in favor of the side s—or perhaps more intuitively, that the factors from W, taken together, provide the court with a reason for deciding for s.

A *standard precedent case* is defined as a standard fact situation together with an outcome and a standard rule through which that outcome is reached or justified. Such a case, then, is a triple of the form $c = \langle X, r, s \rangle$, where X is a fact situation containing the standard factors presented by the case, r is the rule of the case, and s is its outcome. We define three functions—*Facts*, *Rule*, and *Outcome* mapping cases into their component parts, so that, in the case c above, for example, we have $Facts(c) = X$, $Rule(c) = r$, and $Outcome(c) = s$. The concept of a case is subject to two coherence conditions: first, that the rule of the case must actually apply to the underlying fact situation, or equivalently, that the fact situation satisfies the reason that forms the premise of that rule, and second, that the conclusion of the case rule must match the outcome of the case itself. These two coherence conditions can be captured through the general requirements that

$$Facts(c) \models Premise(Rule(c))$$
$$Conclusion(Rule(c)) = Outcome(c)$$

for any case c, or in terms of the particular case displayed above, that $X \models Premise(r)$ and $Conclusion(r) = s$.

These various concepts and constraints can be illustrated through the concrete case $c_1 = \langle X_1, r_1, s_1 \rangle$, where $X_1 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$, with two factors each favoring the plaintiff and the defendant, where r_1 is the rule $\{f_1^\pi\} \rightarrow \pi$, and where the outcome s_1 is π, a decision for the plaintiff. Evidently, the case satisfies our two coherence constraints: the rule of the case is applicable to the facts, and the conclusion of this rule matches the outcome of the case. This particular precedent, then, represents a case in which the court decided for the plaintiff by applying or introducing a rule according to which the presence of the factor f_1^π leads, by default, to a decision for the plaintiff.

Finally, a *standard case base* is defined as a set Γ of standard cases—a set of fact situations presented to various courts, together with their outcomes and the rules justifying these outcomes.

2.2 Two models of constraint

Now, how does an existing case base like this constrain decisions in future cases? We begin by reviewing two models of precedential constraint developed for the standard setting in previous work.

The first is the *result model*, set out in Horty (2004), according to which an existing case base constrains a later court only when that court is presented with an a fortiori fact situation—a situation that is at least as strong for the winning side of some precedent case as the fact situation of that precedent case itself.

Obviously, this model relies on some ordering through which different fact situations can be compared in strength for one side or another. The ordering I propose is one according to which a fact situation Y presents a case for the side s that is at least as strong as that presented by the fact situation X whenever Y contains all the factors from X that support s, and X contains all the factors from Y that support $\bar{s}$, the opposite side. If we let $\leq^s$ represent the strength ordering for the side s, this idea can then be defined formally as follows:

Definition 4 (*Strength for a side: standard*) Let X and Y be standard fact situations. Then Y is at least as strong as X for the side s—written, $X \leq^s Y$—if and only if $X^s \subseteq Y^s$ and $Y^{\bar{s}} \subseteq X^{\bar{s}}$.

This definition, I have argued, conforms to our intuitions, and exhibits a number of plausible formal properties as well. It can be illustrated by considering the previous fact situation $X_1 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$ along with the new situation $X_2 = \{f_1^\pi, f_2^\pi, f_1^\delta\}$.

Here, we have $X_1 \leq^{\pi} X_2$, since X_2 contains all the factors from X_1 that support π, and X_1 contains all the factors from X_2 that support δ; and we can see likewise that $X_2 \leq^{\delta} X_1$.

With this strength ordering $\leq^s$ in place, it is straightforward to define the concept of a fortiori constraint at work in the result model:

Definition 5 (*Result model of constraint*) Let Γ be a case base and X a matching fact situation confronting the court. Then the result model of constraint requires the court to reach a decision in X for the side s if and only if there is some case c from Γ such that $Outcome(c) = s$ and $Facts(c) \leq^s X$.

To continue our example, suppose the background case base is $\Gamma_1 = \{c_1\}$, containing only the previous case c_1, and that the court is currently confronting the situation X_2. Then the result model of constraint requires a decision in this situation for the plaintiff, since $Outcome(c_1) = \pi$ and, as we have seen, $Facts(c_1) \leq^{\pi} X_2$.

The result model presents a picture of precedential constraint that depends only on the comparative strength for a side of the current fact situation relative to the facts of some precedent case, regardless of the explicit rule formulated by the court to justify its decision in that precedent case. There is a long history behind the idea that a court's own efforts at justifying its decision should carry less weight than the decision itself. This history goes back at least to Arthur Goodhart's (1930) thesis that the *ratio decidendi* of a case is determined only by the decision in that case together with its material facts, as identified by the court, and through Goodhart to earlier work by the American legal realists.[5] But, though the history behind this idea may be long, the idea itself represents a minority opinion. Most common law theorists—including Melvin Eisenberg (1988), Joseph Raz (1979), and A.W.B. Simpson (1961)—believe that the reasons offered by the court must be taken seriously as the basis for its decision. This perspective is captured in the *reason model* of constraint, with roots in the work of Grant Lamond (2005), set out precisely in Horty (2011), explored from a philosophical perspective in Horty (2015), and developed in the context of artificial intelligence and law in Horty and Bench-Capon (2012). The central feature of this model is that it makes explicit what is, I feel, implicit in case law: an ordering representing the weight, or priority, of the reasons underlying judicial decisions.

To motivate this new model of constraint, let us return to the case $c_1 = \langle X_1, r_1, s_1 \rangle$—where $X_1 = \{f_1^{\pi}, f_2^{\pi}, f_1^{\delta}, f_2^{\delta}\}$, where $r_1 = \{f_1^{\pi}\} \rightarrow \pi$, and where $s_1 = \pi$—and ask what information is actually carried by this case; what is the court telling us with its decision? Well, two things. With its decision for the plaintiff on the basis of the rule r_1, the court is telling us that the reason for its decision—that is, $Premise(r_1)$, the premise of the rule—carries more weight, or has higher priority, than any reason for the defendant that holds in X_1, the fact situation of the case. And second, if $Premise(r_1)$ itself has higher priority than any reason for the defendant that holds in X_1, the court must also be telling us, at least implicitly, that

5 An interesting discussion of the realist influence on Goodhart is found in Duxbury (2005, pp. 80–90).

any other reason for the plaintiff at least as strong as *Premise*(r_1) must likewise have higher priority than any reason for the defendant that holds in this situation.

We can recall that a reason W for the defendant holds in the situation X_1 whenever $X_1 \models W$, and that a reason Z for the plaintiff is at least as strong as *Premise*(r_1) whenever $Z \Vdash Premise(r_1)$. If we let $<_{c_1}$ represent the priority relation on reasons derived from the particular case c_1, therefore, the force of the court's decision in this case is that: where W is a reason favoring the defendant and Z is a reason favoring the plaintiff, $W <_{c_1} Z$ whenever $X_1 \models W$ and $Z \Vdash Premise(r_1)$. To illustrate: since $\{f_1^\delta\}$ and $\{f_1^\pi, f_3^\pi\}$ are reasons favoring the defendant and the plaintiff respectively, and since we have both $X_1 \models \{f_1^\delta\}$ and $\{f_1^\pi, f_3^\pi\} \Vdash Premise(r_1)$, it follows that $\{f_1^\delta\} <_{c_1} \{f_1^\pi, f_3^\pi\}$—the court's decision in the case c_1 tells us that the reason $\{f_1^\pi, f_3^\pi\}$ favoring the plaintiff must be assigned a higher priority than the reason $\{f_1^\delta\}$ favoring the defendant.

This line of argument leads to the following definition of the priority ordering among reasons derived from a single case:

Definition 6 (*Priority ordering derived from a case*) Let $c = \langle X, r, s \rangle$ be a case, and let W and Z be matching reasons favoring the sides $\bar{s}$ and s respectively. Then the relation $<_c$ representing the priority ordering on reasons derived from the case c is defined by stipulating that $W <_c Z$ if and only if $X \models W$ and $Z \Vdash Premise(r)$.

A reader who is familiar with earlier versions of this definition—set out in Horty (2011) or Horty and Bench-Capon (2012), for example—will notice that I have here replaced the earlier condition that $W \subseteq X$ with the current $X \models W$, and the earlier condition that *Premise*$(r) \subseteq Z$ with the current $Z \Vdash Premise(r_1)$. As noted in the discussion following Definitions 2 and 3, the new conditions are equivalent to the earlier conditions in the standard setting, but will allow the definition to generalize to the dimensional setting as well.

Once we have defined the priority ordering on reasons derived from a single case, we can introduce a priority relation $<_\Gamma$ derived from an entire case base Γ in the natural way, by stipulating that one reason has a higher priority than another according to the case base whenever that priority relation is supported by some particular case from the case base:

Definition 7 (*Priority ordering derived from a case base*) Let Γ be a case base, and let W and Z be matching reasons. Then the relation $<_\Gamma$ representing the priority ordering on reasons derived from the case base Γ is defined by stipulating that $W <_\Gamma Z$ if and only if $W <_c Z$ for some case c from Γ.

And we can then define a case base as consistent as long as it does not provide conflicting information about the priority ordering among reasons—telling us, for some pair of reasons, that each has a higher priority than the other.

Definition 8 (*Consistent case bases*) Let Γ be a case base with $<_\Gamma$ its derived priority ordering. Then Γ is inconsistent if and only if there are matching reasons W and Z such that $W <_\Gamma Z$ and $Z <_\Gamma W$, and consistent otherwise.

Given this notion of consistency, we now turn to the reason model of constraint itself. This model applies, in the first instance, to the rules on the basis of which a court can reach its decision. Here, the intuition is that, in confronting a new situation against the background of an existing case base, the court is required to reach its decision on the basis of a rule that does not introduce inconsistency into that case base.

Definition 9 (*Reason model constraint on rule selection*) Let Γ be a case base and X a matching fact situation confronting the court. Then the reason model of constraint on rule selection requires the court to base its decision on some rule r supporting an outcome s such that the new case base $\Gamma \cup \{\langle X, r, s\rangle\}$ is consistent.

But of course, once this constraint on rule selection is in place, the reason model can naturally be interpreted as requiring a decision for a particular side just in case every rule satisfying the constraint on rule selection supports that side.

Definition 10 (*Reason model constraint on decision*) Let Γ be a case base and X a matching fact situation confronting the court. Then the reason model of constraint on decision requires the court to reach a decision in X for the side s if and only if every rule satisfying the constraint on rule selection supports the side s.

This definition can be illustrated by assuming once more that the background case base is $\Gamma_1 = \{c_1\}$, containing as its single member the familiar case $c_1 = \langle X_1, r_1, s_1\rangle$—where, again, $X_1 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$, where $r_1 = \{f_1^\pi\} \rightarrow \pi$, and where $s_1 = \pi$. Suppose that, against this background, the court confronts the fresh situation $X_3 = \{f_1^\pi, f_1^\delta\}$ and considers finding for the defendant on the basis of $\{f_1^\delta\}$, leading to the decision $c_3 = \langle X_3, r_3, s_3\rangle$, where X_3 is as above, where $r_3 = \{f_1^\delta\} \rightarrow \delta$, and where $s_3 = \delta$. Since $X_3 \models \{f_1^\pi\}$ and $\{f_1^\delta\} \Vdash Premise(r_3)$, we would then have $\{f_1^\pi\} <_{c_3} \{f_1^\delta\}$, according to which the reason $\{f_1^\delta\}$ for the defendant would have to have a higher priority than the reason $\{f_1^\pi\}$ for the plaintiff. But Γ_1 already contains the case c_1, from which we can derive the priority relation $\{f_1^\delta\} <_{c_1} \{f_1^\pi\}$, telling us exactly the opposite. Since the augmented case base $\Gamma_1 \cup \{c_3\}$ would be inconsistent, the reason model constraint on rule selection would prevent the court from deciding the situation X_3 for the defendant on the basis of the rule r_3; and since r_3 is the only rule available supporting the defendant, the reason model constraint on decision for a side therefore requires a decision in this situation for the plaintiff.

It is worth noting that this example illustrates an important feature of our treatment of case base consistency and inconsistency from Definition 8. According to this definition, a case base Γ is inconsistent if there are any two reasons at all each of which is ranked as having a higher priority than the other—any reasons W and Z, that is, such that $W <_\Gamma Z$ and $Z <_\Gamma W$. As it turns out, however, any case base that is inconsistent in this sense will exhibit an inconsistency of a very special sort—an inconsistent ranking between premises of rules from cases belonging to that case base.

Observation 1 Let Γ be a case base with $<_{\Gamma}$ its derived priority relation. Then Γ is inconsistent if and only if there are cases $c = \langle X, r, s \rangle$ and $c' = \langle Y, r', \overline{s} \rangle$ belonging to Γ such that $Premise(r') <_{c} Premise(r)$ and $Premise(r) <_{c'} Premise(r')$.

This result, mentioned here because it will be useful later, has clear computational implications, since it means that, in trying to establish whether a case base is consistent or inconsistent, we need only check the reasons explicitly provided by its case rules against one another, rather than searching through the entire set of reasons supported by the underlying language.

Finally, we turn to the relation between the two models of constraint defined here, the result model and the reason model. It is easy to see, first of all, that constraint according to the result model entails constraint according to the reason model.

Observation 2 Let Γ be a consistent case base and X a new fact situation confronting the court, and suppose the result model of constraint requires a decision for the side s in the situation X. Then the reason model of constraint likewise requires a decision for s in this situation.

What this means, in other words, is that, if the background case base contains a case that was already decided for some side, and the current fact situation is at least as strong for that side as the fact situation from the background case, then it is impossible to find a rule, or a reason, that would consistently support a decision in the current situation for the opposite side.

So result constraint entails reason constraint, but at least in the standard setting, the converse entailment fails. We can see this in the example just considered, where the court is confronting the new fact situation $X_3 = \{f_1^{\pi}, f_1^{\delta}\}$ against the background of the case base $\Gamma_1 = \{c_1\}$. Here, as we saw, the reason model of constraint requires a decision for the plaintiff in the new situation, since a decision for the defendant on the only grounds available would render the background case base inconsistent. But the result model of constraint does not require a decision for the plaintiff, since X_3 is not at least as strong for the plaintiff as the fact situation from some case already decided for the plaintiff—in particular, we do not have $X_1 \leq^{\pi} X_3$.

3 The dimensional setting

3.1 Dimensions and magnitudes

We now move from the standard setting, with a case representation based on standard factors, to the dimensional setting. While a factor is a legally significant proposition, which either holds or does not, but always favors the same side when it does hold, a dimension, we recall, is an ordered set of values, with the ordering corresponding to the extent to which these values favor one side or the other. The importance of dimensions was illustrated earlier with Prakken and Sartor's change of fiscal domicile example, where the issue under dispute is whether a period of residence in a foreign country counts as a change of fiscal domicile, and where one dimension to consider is the duration of that period, with various lengths of time as values and longer lengths of time favoring change of domicile more strongly. Of course, there may be more than one dimension to consider in a given dispute. In the present example, another relevant dimension might be the percentage of the individual's income derived from organizations based in the foreign country, with particular percentages as values and larger percentages favoring change of domicile.

To represent information like this, we start by postulating a set $D = \{d_1, d_2, \ldots, d_n\}$ of dimensions relevant to some area of dispute. For each dimension, we assume an ordered set of values, ranging from those favoring the side s to those favoring the side $\bar{s}$. Where p and q are particular values along some fixed dimension, we take the statement

$$p \preceq^s q$$

to mean that the assignment of the value q to this dimension favors the side s at least as strongly as the assignment of p. The ordering on dimension values is assumed to satisfy the partial-order conditions of reflexivity, transitivity, and antisymmetry

$$\begin{aligned} &p \preceq^s p, \\ &p \preceq^s q \text{ and } q \preceq^s r \text{ implies } p \preceq^s r, \\ &p \preceq^s q \text{ and } q \preceq^s p \text{ implies } p = q, \end{aligned}$$

as well as a duality condition

$$p \preceq^s q \text{ if and only if } q \preceq^{\bar{s}} p,$$

according to which q favors the side s at least as much as p just in case p favors the opposing side $\bar{s}$ at least as much as q.

This notation can be illustrated with our fiscal domicile example if we imagine that the plaintiff is an individual's native country, which is arguing against change of domicile in order to tax the individual's income, and that the defendant is the individual, who is arguing for change of domicile in order to pay, let us suppose, the lower tax rate available in a foreign country. Here, two possible values along the dimension representing the period of residence abroad are six months and eighteen months. If these values are represented simply as 6 and 18, we have $18 \preceq^\pi 6$, since the shorter period abroad favors the plaintiff's argument against change of domicile; duality then tells us that $6 \preceq^\delta 18$, since the longer period abroad favors the individual's argument in favor of change.

Where p is a value along the dimension d, the pair $\langle d, p \rangle$ is a *value assignment*, according to which the dimension d takes on the value p. In contrast to a standard fact situation, defined earlier as a set of standard factors, a *dimensional fact situation*

$$X = \{\langle d,p \rangle : d \in D\}$$

can be defined as a set of values assignments, one for each dimension, subject to the condition that if $\langle d,p \rangle$ and $\langle d,p' \rangle$ both belong to X, then $p = p'$. A dimensional fact situation, in other words, is a function mapping each dimension to a value along that dimension. We take $X(d)$ as the value assigned to the dimension d in the fact situation X, where this idea is defined in the usual way:

$$X(d) = p \text{ if and only if } \langle d,p \rangle \in X.$$

To illustrate: if d_1 and d_2 are the dimensions representing length of time in a foreign country and proportion of income earned from organizations based in that country, then $X_4 = \{\langle d_1, 30 \rangle, \langle d_2, 60 \rangle\}$ is the dimensional fact situation presented by an individual who has spent two and a half years, or thirty months, in a foreign country, while earning sixty percent of his or her income from organizations based in that country. We would therefore have $X_4(d_1) = 30$ and $X_4(d_2) = 60$.

The central conceptual problem presented by the dimensional setting—and emphasized by Bench-Capon—is that, while standard fact situations are constructed out of standard factors, always favoring one side or the other, dimensional fact situations are constructed out of value assignments, which need not, intrinsically, favor any particular side. I address this problem by introducing a different class of factors, like standard factors in possessing a definite polarity, but keyed to the value assignments found in dimensional fact situations.

To motivate the proposal, consider a situation, such as that represented by X_4, in which an individual has been living in a foreign country for two and a half years, and imagine that the question of fiscal domicile hinges on whether or not a period of that length counts as a long duration. How could a court reach a decision in this situation, and how could it justify its decision? My suggestion is that the court might focus on some particular value along that dimension—a *reference value*—and then both reach and justify its decision by comparing the value of the dimension in the current fact situation to that reference value. Suppose, for instance, that the value of one year seems, to the court, like a sufficient length of time to count as a long duration. Taking this value as a reference value, the court could then register its decision in the current situation by ruling for change of fiscal domicile, and so in favor of the defendant, on the grounds that the period of residence in the foreign country lasted at least a year.

This proposition—to spell it out, that the actual period of residence abroad favors the defendant at least as much as a period of one year—is a kind of factor: it either holds or does not hold in any fact situation, and always favors the same side, the defendant, when it does hold. Generalizing from our example, then, where p is some value along the dimension d, we now introduce the concept of a *magnitude factor favoring the side s*, as a statement of the form

$$M^s_{d,p}$$

carrying the meaning that: the actual value assigned to the dimension d favors the side s as least as strongly as the reference value p. Continuing to take d_1 as the dimension representing length of time abroad, then, the magnitude factor at work in our example can be expressed as $M^{\delta}_{d_1,12}$, the proposition that the actual value assigned to d_1 in the situation at hand favors the defendant at least as much as a value of twelve months, or one year.

Once these magnitude factors have been introduced, we can now, following the pattern from the standard setting, define a *magnitude reason favoring the side s* as a set of magnitude factors favoring that side. A factor collection of the form $\{M^{\pi}_{d_1,p}, M^{\pi}_{d_2,q}\}$, then, would be a magnitude reason favoring the plaintiff, carrying the conjunctive meaning that the actual value assigned to d_1 favors the plaintiff as least as strongly as p and the actual value assigned to d_2 favors the plaintiff as least as strongly as q. A collection the form $\{M^{\pi}_{d_1,p}, M^{\delta}_{d_2,q}\}$, on the other hand, would not be a reason at all, since the two magnitude factors it contains favor different sides. As before, reasons containing only a single factor are identified in meaning with the factor they contain.

There is one oddity that deserves mention. In our motivating example, the factor $M^{\delta}_{d_1,12}$—that the individual has spent at least a year abroad—does seem to represent a real consideration in favor of the defendant, since a year, as a reference value, is a significant length of time. But what if the factor were keyed to a less significant reference value, such as a single month, or even a single day? At least in our initial discussion, we propose no formal restrictions bearing on the intuitive significance of the reference point: any value on the scale will do.[6] But is this sensible? Can we really suppose that the proposition that an individual has spent even a single day abroad should count as a factor, or a reason, favoring a judgment for change of fiscal domicile? To this question there is a simple response. To say that a consideration favors a particular side does not necessarily mean that it favors that side very strongly. We can allow that the proposition that an individual has spent at least a day abroad counts as a reason supporting change of fiscal domicile—after all, the proposition rules out periods of less than a day, and holds in all longer periods—while still allowing that it is an exceptionally weak reason, likely to be outweighed by just about any serious reason favoring the other side.[7]

Now, what about satisfaction and entailment? In the standard setting, where fact situations were simply sets of standard factors, a fact situation could be said to satisfy a factor whenever that factor belonged to the situation. But this idea, set out in Definition 1, cannot carry over to the dimensional setting, since a fact situation is now defined as a set of value assignments and a factor is something else entirely—a statement of the form $M^{s}_{d,p}$, carrying the meaning, once again, that the value assigned to dimension d favors the side s at least as strongly as the value p. Here, since the value actually assigned to the dimension d in some situation X is simply $X(d)$, and since this value favors the side s at least as strongly as the value p whenever $p \preceq^{s} X(d)$, the conditions under which a dimensional fact situation satisfies a magnitude factor can be defined as follows:

[6] Later, in Sect. 4, we discuss restricting reference points to salient values on the dimensional scale.

[7] I owe this form of argument, or perhaps this argument itself, to Schroeder (2007).

Definition 11 (*Factor satisfaction: dimensional*) Where X is a dimensional fact situation and $M^s_{d,p}$ is a magnitude factor, X satisfies $M^s_{d,p}$—written, $X \models M^s_{d,p}$—if and only if $p \preceq^s X(d)$.

Returning to the situation $X_4 = \{\langle d_1, 30\rangle, \langle d_2, 60\rangle\}$, we can now verify that $X_4 \models M^{\delta}_{d_1,12}$. As we noted earlier, $X_4(d_1) = 30$, and since a period abroad of thirty months favors the conclusion of an absence of long duration, and so the defendant, at least as much as a period of twelve months, we therefore have $12 \preceq^{\delta} X_4(d_1)$.

Once the previous treatment of factor satisfaction from Definition 1 has been replaced with this new treatment, from Definition 11, our earlier definitions of reason satisfaction and entailment can be carried over without change to the dimensional setting. We can say, in accord with Definition 2, that a dimensional fact situation satisfies a magnitude reason whenever that fact situation satisfies each magnitude factor belonging to that reason, and in accord with Definition 3, that one magnitude reason entails another whenever every dimensional fact situation that satisfies the first also satisfies the second.

Comments are again called for, corresponding to those following Definitions 2 and 3. First, in applying these same definitions in both the standard and the dimensional setting, we must now remember the requirement that fact situations and reasons should match—where standard fact situations match standard reasons and dimensional fact situations match magnitude reasons. Thus, only dimensional fact situations can be said to satisfy magnitude reasons, and only standard fact situations can be said to satisfy standard reasons; it makes no sense to ask, for example, whether a dimensional fact situation satisfies a standard reason. Likewise, the entailment relation holds between magnitude reasons, or between standard reasons, but not between standard and magnitude reasons.

Second, we noted earlier that, in the standard setting, the concepts of reason satisfaction and reason entailment were both equivalent to set inclusions—a standard fact situation X satisfies a standard reason W just in case $W \subseteq X$, and a standard reason W entails a standard reason Z just in case $Z \subseteq W$. But neither of these relations can be reduced to simple inclusions in the dimensional setting. A dimensional fact situation is never a superset of a magnitude reason it satisfies, since dimensional fact situations and magnitude reasons contain different kinds of things. And where W and Z are magnitude reasons, although it does hold that, if $Z \subseteq W$, then $W \Vdash Z$, the converse fails. To see this, we can note that residing in a foreign country for at least two and a half years and residing in a foreign country for at least one year are two different reasons supporting the conclusion of a lengthy stay abroad—represented as $\{M^{\delta}_{d_1,30}\}$ and $\{M^{\delta}_{d_1,12}\}$ respectively—but we have $\{M^{\delta}_{d_1,30}\} \Vdash \{M^{\delta}_{d_1,12}\}$ without $\{M^{\delta}_{d_1,12}\} \subseteq \{M^{\delta}_{d_1,30}\}$.

Finally, in the dimensional setting, as before, logical entailment continues to correspond to a relation of strength for a side among reasons. For example, because $\{M^{\delta}_{d_1,30}\} \Vdash \{M^{\delta}_{d_1,12}\}$, we know that $\{M^{\delta}_{d_1,30}\}$ is a stronger reason favoring the defendant than $\{M^{\delta}_{d_1,12}\}$—again, a period of thirty months abroad favors the defendant at least as much as a period of twelve months abroad.

Moving on, then: If W is a magnitude reason favoring the side s, we can now define $W \rightarrow s$ as a *magnitude rule*, where this rule is interpreted defeasibly, just like a standard rule, and where the functions *Premise* and *Conclusion* picking out the premise and conclusion of this rule are defined as before. Following the pattern of the standard setting, we can define a *dimensional case* as a triple $c = \langle X, r, s\rangle$, where X is a dimensional fact situation, r is a magnitude rule justifying a particular outcome, and s is the case outcome itself. As before, we have three functions—*Facts*, *Rule*, and *Outcome*—mapping cases into their component parts. And we again require, as a coherence condition on the concept of a case, both that the rule of the case should apply to its fact situation and that the conclusion of the case rule should match the case outcome, so that, with c as above, $X \models Premise(r)$ and $Conclusion(r) = s$.

These ideas can be illustrated in the case $c_4 = \langle X_4, r_4, s_4\rangle$, where the familiar $X_4 = \{\langle d_1, 30\rangle, \langle d_2, 60\rangle\}$ is the underlying dimensional fact situation, where r_4 is the magnitude rule $\{M^{\delta}_{d_1,12}\} \rightarrow \delta$, and where s_4 is δ, a decision for the plaintiff. The case, then, is one in which, confronted with an individual who has spent two and a half years in a foreign country and during that period earned sixty percent of his or her income from organizations based in that country, the court ruled for change of fiscal domicile, and so in favor of the defendant, on the grounds that the individual spent at least a year abroad. Note that this case satisfies both of our coherence conditions, since the premise of the case rule holds in the fact situation of the case and the conclusion of this rule coincides with the case outcome.

Finally, and as in the standard setting, we define a *dimensional case base* Γ as a set of dimensional cases.

3.2 Constraint

How can our two models of constraint, result and reason, be adapted to the dimensional setting?

The result model, we recall, was meant to capture a fortiori reasoning—according to which a court is constrained to decide a new fact situation for a particular side whenever the new situation is at least as strong for that side as the fact situation from a case that has already been decided for that side—and so depends on an ordering through which different fact situations can be compared in strength for one side or another. In the standard setting, with standard fact situations built from standard factors, this ordering was set out in Definition 4, but of course, that definition is no longer applicable in the dimensional setting. Fortunately, it is plain how the new definition should go: the dimensional fact situation Y should now be defined to be at least as strong as the dimensional fact situation X whenever the value of Y is at least as strong as the value of X along every dimension. Continuing to use $\leq^s$ to represent strength for a side s, this new definition can be stated formally as follows:

Definition 12 (*Strength for a side: dimensional*) Let X and Y be dimensional fact situations. Then Y is at least as strong as X for the side s—written, $X \preceq^{s} Y$—if and only if $X(d) \preceq^{s} Y(d)$ for each dimension d from D.

And once this new concept of strength for a side is in place, our previous specification of the result model in terms of strength for a side, set out in Definition 5, can be carried over without change, attending only to matching restrictions.

This model can be illustrated in the dimensional setting by taking as background the dimensional case base $\Gamma_2 = \{c_4\}$, containing only the earlier dimensional case c_4, and imagining that the court is currently confronting the new situation $X_5 = \{\langle d_1, 36\rangle, \langle d_2, 65\rangle\}$, representing a state of affairs in which an individual spent three years in a foreign country while earning sixty-five percent of his or her income from organizations based there. Comparing this fresh situation to the earlier $X_4 = \{\langle d_1, 30\rangle, \langle d_2, 60\rangle\}$, our new Definition 12 tells us that $X_4 \preceq^{\delta} X_5$, since $30 \preceq^{\delta} 36$ along the dimension d_1 and $60 \preceq^{\delta} 65$ along the dimension d_2—a longer period in the foreign country favors the defendant, and so does a greater proportion of income earned from foreign organizations. Definition 5 then tells us that the result model of constraint requires a decision for the defendant in the situation X_5, since this new situation is at least as strong for the defendant as X_4, the fact situation from a previous case c_4 that was already decided for the defendant.

The result model, then, can be adapted in a straightforward way to the dimensional setting, but what about the reason model? Here, we can see the point of our earlier reformulation of the reason model in terms of the logical ideas of reason satisfaction and reason entailment—for it turns out that, subject only to matching restrictions, the treatment of the reason model set out earlier carries over without change to the dimensional setting.

To spell it out: According to Definitions 9 and 10, the reason model requires a court, faced with a fresh fact situation and working against the background of an existing case base, to reach a decision that maintains consistency of that case base. A case base is consistent, according to Definition 8, as long as there are no two reasons each of which is prioritized over the other on the basis of the priority ordering derived from that case base, where this idea is set out in Definition 7, which itself relies on the central concept, from Definition 6, of the priority ordering on reasons derived from a single case. This latter definition draws on the ideas of reason satisfaction and reason entailment from Definitions 2 and 3, which themselves bottom out, in the standard setting, in the treatment of standard factor satisfaction from Definition 1 and, in the dimensional setting, in the new treatment of magnitude factor satisfaction from Definition 11. The entire structure of the reason model is thus identical in the standard and dimensional settings, differing only at the very bottom, with different treatments of standard and magnitude factors.

Still, even though the dimensional reason model simply parallels the standard version, it is worth discussing a few examples in order to understand the shape of this model in the more complex dimensional setting.

Example 1 We take as background the previous case base $\Gamma_2 = \{c_4\}$, containing $c_4 = \langle X_4, r_4, s_4 \rangle$, where $X_4 = \{\langle d_1, 30\rangle, \langle d_2, 60\rangle\}$, where $r_4 = \{M^{\delta}_{d_1,12}\} \rightarrow \delta$, and where $s_4 = \delta$. This case, once again, represents a situation in which the defendant spent two and a half years in a foreign country while earning sixty percent of his or her income from organizations based there, and in which the court decided for the defendant on the grounds that the period abroad lasted at least a year. Now, against this background, imagine that a new court confronts the new fact situation $X_6 = \{\langle d_1, 36\rangle, \langle d_2, 10\rangle\}$, representing an individual who easily satisfies the rule of the c_4 court by spending three years in a foreign country, but during that period earned only ten percent of his or her income there.

In considering this situation, the new court might be struck by the remarkably low proportion of income earned from foreign organizations and suppose that this possibility had not been foreseen by the c_4 court when it formulated its rule based solely on length of stay abroad. The new court might, therefore, hope to distinguish on this basis, ruling against change of domicile, and so in favor of the plaintiff, on the grounds that no more than, say, twenty-five percent of income was earned from foreign organizations. This decision would be represented by the case $c_6 = \langle X_6, r_6, s_6 \rangle$, where X_6 is as above, where $r_6 = \{M^{\pi}_{d_2,25}\} \rightarrow \pi$, and where $s_6 = \pi$.

Can the new court rule as it prefers? It can, according to the reason model, since the resulting case base $\Gamma_2 \cup \{c_6\}$ is consistent. This fact can be verified as follows: Suppose $\Gamma_2 \cup \{c_6\}$ were inconsistent. Since the case base contains only two cases, c_4 and c_6, it would then follow by Observation 1 that both $Premise(r_6) <_{c_4} Premise(r_4)$ and $Premise(r_4) <_{c_6} Premise(r_6)$. By Definition 6, the first of these priority relations requires that $X_4 \models Premise(r_6)$, which holds just in case $25 \preceq^{\pi} X_4(d_2)$ by Definitions 2 and 11, the evaluation rules for reasons and magnitude factors. From this we have $25 \preceq^{\pi} 60$, since $X_4(d_2) = 60$. Our domain assumptions, however, tell us that $60 \preceq^{\pi} 25$, since a smaller proportion of income earned from foreign organizations favors the plaintiff as least as much as a larger proportion. And then antisymmetry of the $\preceq^{\pi}$ relation tells us that $25 = 60$, which is false.

This first example presents a form of distinguishing that is, in many ways, canonical. Even though the new situation X_6 is covered by the previous rule r_4, the court notices that this new situation is significantly weaker than the situation X_4, in which that previous rule had been formulated, along a dimension that had not been addressed by that rule. The court can therefore imagine that this dimension of weakness had not been anticipated in the formulation of the previous r_4, and distinguish accordingly.

Example 2 In the same situation, confronting $X_6 = \{\langle d_1, 36\rangle, \langle d_2, 10\rangle\}$ against the background of $\Gamma_2 = \{c_4\}$, the new court might also advance a more nuanced decision, still holding that foreign income of twenty-five percent or less presents a stronger reason for the plaintiff than a period abroad of three years or more does for the defendant, but allowing that, even this low a proportion of income could itself be outweighed by a much longer period abroad—say, five years, or sixty months. A court reasoning along these lines could justify its decision on the grounds that the defendant both spent no longer than five years abroad and also earned no more than twenty-five percent of his or her income from foreign organizations. A decision like this, with a magnitude rule containing magnitude factors from different dimensions, could be represented by the case $c_7 = \langle X_7, r_7, s_7 \rangle$, where $X_7 = X_6$, where $r_7 = \{M^{\pi}_{d_1,60}, M^{\pi}_{d_2,25}\} \rightarrow \pi$, and where $s_7 = \pi$. We leave it to the reader to verify that $\Gamma_2 \cup \{c_7\}$ is consistent, so that this decision is likewise permitted by the reason model.

Example 3 Suppose, however, that against the same background of $\Gamma_2 = \{c_4\}$, the same situation $X_6 = \{\langle d_1, 36 \rangle, \langle d_2, 10 \rangle\}$ now comes before a court that evaluates change of fiscal domicile cases against very high standards for proportion of income earned abroad. Imagine that this court adopts the extreme position that, in order for a period abroad by some individual is to count as a change of domicile, more than seventy-five percent of the individual's income during that period should have been earned from foreign organizations. A court like this would prefer to rule against change of domicile in X_6, and so in favor of the plaintiff, on the grounds that the individual fails the seventy-five percent test. This decision would be represented by the case $c_8 = \langle X_8, r_8, s_8 \rangle$, where $X_8 = X_6$, where $r_8 = \{M^{\pi}_{d_2,75}\} \rightarrow \pi$, and where $s_8 = \pi$.

Again we ask whether the court can rule as it prefers, and the answer this time is that it cannot, since the resulting case base $\Gamma_2 \cup \{c_8\}$ would now be inconsistent. To see this, we note that the pair of cases c_4 and c_8 belonging to this case base would generate an inconsistent ordering on the premises of its case rules, since each of these rules would then hold in the fact situation of the other case, where the other rule was preferred. More precisely, we would have $X_4 \models Premise(r_8)$ and of course $Premise(r_4) \Vdash Premise(r_4)$, from which it follows by Definition 6 that $Premise(r_8) <_{c_4} Premise(r_4)$, but we would also have $X_8 \models Premise(r_4)$ and $Premise(r_8) \Vdash Premise(r_8)$, from which it follows that $Premise(r_4) <_{c_8} Premise(r_8)$.

Just as in Example 1, the court hopes to distinguish here on the basis of weakness along a dimension that had not been addressed by the previous rule r_4, but this time, inconsistency results, because the new rule r_8 applies to the fact situation X_4 in which the previous rule had been formulated.

Example 4 We now turn to a slightly different kind of example. Still working against the background of $\Gamma_2 = \{c_4\}$, we now consider the new situation $X_9 = \{\langle d_1, 15 \rangle, \langle d_2, 75 \rangle\}$, representing an individual who spent fifteen months abroad while earning seventy-five percent of his or her income from foreign organizations. Compared to the individual from the original situation X_4, this individual has earned a greater proportion of income from foreign organizations but has spent less time abroad, though still enough to satisfy the rule of the c_4 court, which requires only twelve months abroad to justify a decision for change of domicile. Imagine, however, that this new situation comes before a court that cares little about proportion of income earned from foreign organizations, but applies stricter standards for length of time abroad. The new court would prefer to rule against change of domicile in this situation, and so for the plaintiff, on the grounds that the defendant has spent less than two years abroad—that is, that the actual period abroad favors the plaintiff at least as much as a period of two years, or twenty-four months. The resulting decision would be represented by the new case $c_9 = \langle X_9, r_9, s_9 \rangle$, where X_9 is as above, where $r_9 = \{M^{\pi}_{d_1,24}\} \rightarrow \pi$, and where $s_9 = \pi$.

Can the new court rule as it prefers? It can, according to the reason model, since the resulting case base $\Gamma_2 \cup \{c_9\}$ is consistent. The argument is similar to that from Example 1, but we review it here, since the examples are slightly different. Suppose $\Gamma_2 \cup \{c_9\}$ were inconsistent. By Observation 1, this would imply, since the case base contains only the two cases c_4 and c_9, that we must have both $Premise(r_9) <_{c_4} Premise(r_4)$ and $Premise(r_4) <_{c_9} Premise(r_9)$. The first of these priority relations requires that $X_4 \models Premise(r_9)$, which holds just in case $24 \preceq^{\pi} X_4(d_1)$. From this we can conclude that $24 \preceq^{\pi} 30$, since $X_4(d_1) = 30$. Our domain assumptions, however, tell us that $30 \preceq^{\pi} 24$, since a shorter period abroad favors the plaintiff at least as much as a longer period. And then antisymmetry of the $\preceq^{\pi}$ relation tells us that $24 = 30$, which is false.

The c_9 decision, then, is permitted by the reason model, since this decision is consistent with the earlier c_4. Of course, there is, quite plainly, a disagreement of sorts between the c_4 and c_9 courts, since the c_4 court bases its decision on the rule that any period of at least a year abroad is sufficient to justify change of fiscal domicile, while the c_9 court relies on the rule that there is no change of domicile as long as the period abroad is two years or less. But according to the reason model, a disagreement like this does not rise to the level of inconsistency. Inconsistency results only when a new situation is decided on the basis of a rule which both applies to a previous case and yields a result different from that reached in the previous case. And here, the new rule r_9 fails to apply to the fact situation from c_4, the only previous case in the case base.

4 Collapse and recovery

4.1 Collapse

Both Observations 1 and 2 continue to hold in the dimensional setting, though the second requires a fresh proof in this new setting.

What Observation 2 tells us, of course, is that result constraint entails reason constraint. And as we saw in the discussion following that observation, the converse does not hold in the standard setting: a court can be required by the reason model to reach a decision for a particular side even though that decision is not forced by the result model. Once we move to the full dimensional setting, however, it turns out that the converse of Observation 2 holds as well: not only does result constraint entail reason constraint, but reason constraint entails result constraint—the two models of constraint, which are distinct in the standard setting, now collapse into one.

Observation 3 Let Γ be a consistent dimensional case base and X a new dimensional fact situation confronting the court, and suppose the reason model of constraint requires a decision for the side s in the situation X. Then the result model of constraint likewise requires a decision for the side s in this situation.

This observation will be verified formally in the Appendix, but is worth illustrating here, with an example that contains the germ of the full proof.

Example 5 Again working against the background of the case base $\Gamma_2 = \{c_4\}$—containing $c_4 = \langle X_4, r_4, s_4 \rangle$, where $X_4 = \{\langle d_1, 30\rangle, \langle d_2, 60\rangle\}$, where $r_4 = \{M^{\delta}_{d_1,12}\} \rightarrow \delta$, and where $s_4 = \delta$—we now suppose the court confronts the new situation $X_{10} = \{\langle d_1, 29\rangle, \langle d_2, 75\rangle\}$. This new situation is similar to the situation $X_9 = \{\langle d_1, 15\rangle, \langle d_2, 75\rangle\}$ from Example 4. Compared to the fact situation from c_4, the single case from the background case base, both X_9 and X_{10} are considerably stronger for the defendant along the dimension d_2, proportion of income earned abroad. In contrast to X_9, however, which is considerably weaker for the defendant along the dimension d_1, length of time abroad, the new X_{10} is only very slightly weaker for the defendant along this dimension—here, the individual spent twenty-nine months abroad rather than thirty, a difference of only a single month.

Still, since there is some dimension along which X_{10} is weaker than the fact situation from c_4, this case cannot form the basis of an a fortiori argument for the defendant in this new situation. A decision for the defendant is not, therefore, required by the result model, and what Observation 3 tells us is that a decision for the defendant cannot then be required by the reason model either—there must be some rule on the basis of which the court can consistently decide for the plaintiff. How do we construct such a rule? As it turns out, if X_{10} is weaker along any dimension than the fact situation from c_4, then this weakness—however slight, however limited in extent—can be used to form a rule consistently supporting the plaintiff. In this case, then, there is nothing to prevent the court from ruling for the plaintiff on the basis of the factor $M^{\pi}_{d_1,29}$—that the individual spent no more than twenty-nine months abroad. The resulting decision would be represented by the case $c_{10} = \langle X_{10}, r_{10}, s_{10} \rangle$, where $X_{10} = X_6$, where $r_{10} = \{M^{\pi}_{d_1,29}\} \rightarrow \pi$, and where $s_{10} = \pi$. The reader can verify that the expanded case base $\Gamma_2 \cup \{c_{10}\}$ is consistent, so that this decision is allowed by the reason model, by adapting the argument from Example 4.

4.2 Two routes toward recovery

The collapse of the reason model of constraint into the result model is surprising and, in some ways, disturbing. It is, as we have seen, the accepted view among legal theorists that the reasons offered by a court to justify its decisions must be taken seriously as an element influencing precedential constraint—and it simply seems like good sense to suppose that it should matter why decisions were made, not just what decisions were made. But at least if the reason model is correct, as a way of understanding the force of the reasons underlying a decision, then none of this can be right. In that case, what the collapse of the reason model shows is that, in the dimensional setting, it makes no difference at all how courts attempt to justify their decisions, what reasons they offer. All that matters is the decision itself, along with the strength of the underlying fact situation for a particular side—there is nothing to precedential constraint that goes beyond simple a fortiori reasoning.

I do not think we should accept this pessimistic outcome. Instead, I believe the conclusion to draw from the collapse is that the reason model as developed so far, although perhaps adequate for the standard setting, needs to be refined before it can be applied in the richer dimensional setting. We need to reconsider the conditions under which rules can legitimately be distinguished—the conditions, that is, under which a rule that was earlier thought to justify a particular outcome, and that applies to the present case, can now be overridden by some other rule in favor of the opposite outcome. As it currently stands, the reason model is based on the idea that the earlier rule can now be overridden by any new rule whatsoever, as long as the new decision justified by that rule is consistent with the existing case base. What the collapse of the reason model shows, however, is that mere consistency is not enough: further restrictions must be added.

Without meaning to suggest that these options exhaust the possibilities, I will sketch here two ways of refining the reason model of constraint, each of which is sufficient to reestablish a distinction between the reason model and the result model.

The first is based on the idea that an earlier rule can be overridden, not by any new rule whatsoever, subject merely to consistency, but only by a rule that addresses dimensions that are different from those addressed by the earlier rule. To formulate this suggestion precisely, let us say that a rule r *addresses* a dimension d just in case the premise $Premise(r)$ of that rule contains a magnitude factor of the form $M^s_{d,p}$, and then that r is *separated* from a rule r' just in case the sets of dimension addressed by r and r' do not overlap. And let us also extend the existing function *Rule* from individual cases to case bases in the natural way, so that $Rule(\Gamma) = \{Rule(c) : c \in \Gamma\}$ is the set of rules contained in cases belonging to the case base Γ. We can then arrive at our first refinement of the reason model by replacing our original constraint on rule selection, from Definition 9, with a new constraint according to which, in confronting a new situation, the court is required to base its decision on a rule that is, not only consistent with the existing case base, but also separated from any rule from the existing case base that both applies to the current situation and supports the opposite outcome.

Definition 13 (*Reason model constraint on rule selection, first refinement*) Let Γ be a dimensional case base and X a matching fact situation confronting the court. Then the reason model of constraint on rule selection requires the court to base its decision on some rule r supporting an outcome s such that (1) $\Gamma \cup \{\langle X, r, s\rangle\}$ is consistent, and (2) r is separated from each rule r' from $Rule(\Gamma)$ such that $X \models Premise(r')$ and $Outcome(r') = \overline{s}$.

This refinement of the reason model can be illustrated by comparing the previous Examples 1 and 4, both of which take as background the previous case base $\Gamma_2 = \{c_4\}$, containing $c_4 = \langle X_4, r_4, s_4\rangle$, where $X_4 = \{\langle d_1, 30\rangle, \langle d_2, 60\rangle\}$, where $r_4 = \{M^{\delta}_{d_1,12}\} \rightarrow \delta$, and where $s_4 = \delta$. In Example 1, even though the earlier r_4 applies to a new situation $X_6 = \{\langle d_1, 36\rangle, \langle d_2, 10\rangle\}$, our original version of the reason model allows the court to decide for the plaintiff in this situation on the basis of a new rule $r_6 = \{M^{\pi}_{d_2,25}\} \rightarrow \pi$, since the resulting decision is consistent. The rule r_6 thus satisfies the first clause of Definition 13—the consistency clause—and as we can see, it satisfies the second clause as well, since the new r_6 address the dimension d_2 while the earlier r_4 addresses the separate dimension d_1. As a result, our refined version of the reason model likewise allows the court to decide the situation X_6 on the basis of this new rule. In Example 4, by contrast, we again confront a case in which the earlier r_4 applies to a new situation $X_9 = \{\langle d_1, 15\rangle, \langle d_2, 75\rangle\}$, and in which our original version of the reason model allows the court to decide this situation for the plaintiff on the basis of a new rule $r_9 = \{M^{\pi}_{d_1,24}\} \rightarrow \pi$, since the resulting decision is consistent. Again, then, the new rule r_9 satisfies the first clause of Definition 13, but in this case r_9 fails to satisfy the second clause, since the earlier r_4 both applies in the new situation and supports an opposing outcome, yet the new r_9 is not separated from the earlier r_4—both rules address the dimension d_1. As a result, while a decision on the basis of r_9 is allowed by the original version of the reason model, it is not allowed by our first refinement.

Example 4 can also be used to establish that constraint under the refined version of the reason model does not entail constraint under the result model, so that collapse is avoided. To see this, we first show as follows that, in this example, the refined reason model requires a decision for the defendant in the situation X_9. Suppose otherwise, that the refined reason model allows a decision for the plaintiff. Then there is a rule r supporting the plaintiff that satisfies both clauses of Definition 13—that is, a rule r such that (1) $\Gamma_2 \cup \{c\}$ is consistent where $c = \langle X_9, r, \pi\rangle$, and (2) r is separated from the earlier r_4. Since r_4 addresses the dimension d_1, separation tells us that r must address d_2, from which it follows that each magnitude factor from $Premise(r)$ has the form $M^{\pi}_{d_2,p}$. Where $M^{\pi}_{d_2,p}$ is an arbitrary such factor, since $X_9 \models Premise(r)$, we know that $X_9 \models M^{\pi}_{d_2,p}$, from which we have $p \leq^{\pi} X_9(d_2)$, or $p \leq^{\pi} 75$ since $X_9(d_2) = 75$. But of course, $75 \leq^{\pi} 60$ by the nature of the d_2 dimension, so that $p \leq^{\pi} 60$ by transitivity. From this it follows that $p \leq^{\pi} X_4(d_2)$, since $X_4(d_2) = 60$, so that $X_4 \models M^{\pi}_{d_2,p}$. Since the particular factor $M^{\pi}_{d_2,p}$ was arbitrary, we have $X_4 \models Premise(r)$, but we also have $X_9 \models Premise(r_4)$ since the earlier rule applies to the new situation. These two facts, together with the obvious $Premise(r_4) \Vdash\!\dashv Premise(r_4)$ and $Premise(r) \Vdash\!\dashv Premise(r)$, allow us to conclude that both $Premise(r_4) <_c Premise(r)$ and $Premise(r) <_{c_4} Premise(r_4)$. From this, it follows that $\Gamma_2 \cup \{c\}$ cannot be consistent and our assumption fails: there is no rule supporting the plaintiff and satisfying the new Definition 13, and so our first refinement of the reason model requires a decision for the defendant in X_9. But it is easy to see that the result model of constraint does not require a decision for the defendant, since the only case belonging to Γ_2 is c_4 and we do not have $X_4(d) \leq^{\delta} X_9(d)$ for each dimension d—in particular, we do not have $X_4(d_2) \leq^{\delta} X_9(d_2)$.

A note: Although this first refinement of the reason model is promising in many ways, it is not entirely unproblematic. The challenge involved in developing an account of precedential constraint lies in balancing the constraints imposed by past decisions with the freedom to respond in creative ways to new situations. And it might seem that, in requiring that the rules formulated in earlier cases can be overridden only on the basis of new rules that are separated from those earlier rules, this first refinement of the reason model is too restrictive of our freedom to respond to new situations. Consider a different sort of example, in which there is only one relevant dimension. Imagine that the issue under consideration, for an ordinary speaker of English, is whether individuals are, or are not, to be classified as tall—a judgment based entirely on the value registered by those individuals along the dimension of height. Suppose the first individual considered by the speaker is, say, Kobe Bryant, the 6 foot 6 inch former professional basketball player. And suppose that, confronted with Bryant, the speaker decides the he is indeed tall, offering as justification the rule: "Anyone over 6 feet in height is tall". This decision is like a precedent case. A judgment is reached in a particular situation on the basis of a rule that applies more widely, not just to the situation at hand.

Now suppose the next individual to be considered is another former basketball player, Isiah Thomas. Although Thomas is a healthy 6 feet 1 inches in height, I think it is not implausible that the speaker might wish to decide—when actually confronted with Thomas, in light of that felt experience—that an individual of that height is not, in fact, particularly tall. Even though the previous rule, announced in the case of Bryant, applies to Thomas, the original version of the reason model allows the speaker the freedom, in this case, to respect her felt experience and decide that Thomas is not tall, perhaps on the basis of the new rule: "Anyone under 6 feet 3 inches in height is not tall". But our initial reformulation of the reason model denies that the speaker has this freedom, since the new rule is not separated from the original, but addresses the same dimension. Indeed, in situations like this, where there is only one dimension of concern, the rules justifying decisions must be taken, according to this reformulation, as strict rules, generalizations that are not subject to exceptions.

A different refinement of the reason model can be arrived at by replacing our original constraint on rule selection with a constraint according to which, in confronting a new situation, the court is required to base its decision on a rule that is consistent with the existing case base but also—to choose a word—*acceptable*.

Definition 14 (*Reason model constraint on rule selection, second refinement*) Let Γ be a dimensional case base and X a matching fact situation confronting the court. Then the reason model of constraint on rule selection requires the court to base its decision on some rule r supporting an outcome s such that (1) $\Gamma \cup \{\langle X, r, s \rangle\}$ is consistent, and (2) r is acceptable.

Evidently, this new definition provides not so much a constraint as a constraint schema, allowing for a variety of interpretations of the notion of acceptability. To take an extreme example, we might declare as acceptable exactly those rules that are separated from the existing rules that apply to the current situation and support opposing results, in which case this refinement would simply coincide with that suggested earlier. But there are also more substantive notions of acceptability available—the acceptable rules might be those that promote the right values, or those that are coherent with the existing set of rules in a way that goes beyond mere consistency.[8]

As these examples show, the idea of acceptability can be understood in a variety of ways. But just for the sake of illustration, let us suppose that the acceptable rules are those that are based on the *salient* values on a dimensional scale, where the notion of salience, along the two dimensions from our current domain, is specified as follows:

> Along the dimension d_1, representing length of time abroad, the salient values are those based on intervals of one year, so that the values of one year, two years, three years, and so on, are salient.
>
> Along the dimension d_2, representing proportion of income earned abroad, the salient values are those based on intervals of twenty-five percent, so that the values of twenty-five percent, fifty percent, seventy-five percent, and one hundred percent are salient.

These definitions tell us, for example, that the rule $\{M^{\delta}_{d_1,24}\} \to \delta$, according to which an individual changed domicile because she spent at least two years abroad, is acceptable, since two years is a salient length of time; but the rule $\{M^{\pi}_{d_1,19}\} \to \pi$, according to which the individual failed to change domicile because she spent nineteen months or less abroad, is not acceptable, since a length of nineteen months is not salient. In the same way, the rule $\{M^{\delta}_{d_2,50}\} \to \delta$, according to which an individual changed domicile because she earned at least fifty percent of her income abroad, is acceptable, since fifty percent is a salient proportion of income; but the rule $\{M^{\pi}_{d_2,37}\} \to \pi$, according to which the individual failed to change domicile because she earned no more than thirty-seven percent of income abroad, is not acceptable, since thirty-seven percent of income is not a salient value.

Based on this understanding of salience, and so acceptability, we can now consider a final example to show that constraint under this second refinement of the reason model does not entail constraint under the result model, so that, again, collapse is avoided.

Example 6 Continuing to work against the background of the case base $\Gamma_2 = \{c_4\}$—containing $c_4 = \langle X_4, r_4, s_4\rangle$, where $X_4 = \{\langle d_1, 30\rangle, \langle d_2, 60\rangle\}$, where

[8] For an emphasis on coherence as a criterion of rule acceptability, see, of course, Dworkin (1977) and Dworkin (1986).

$r_4 = \{M^{\delta}_{d_1,12}\} \rightarrow \delta$, and where $s_4 = \delta$—let us now suppose that the court confronts the new situation $X_{11} = \{\langle d_1, 29\rangle, \langle d_2, 59\rangle\}$. This new situation is just slightly weaker than the fact situation from the original c_4 along each dimension. But, since it is indeed weaker, the original c_4 cannot be used as the basis of an a fortiori argument for the defendant in the new situation, and so the result model does not require a decision for the defendant. On the other hand, the reason model as refined in Definition 14, with acceptability understood as salience, does require a decision for the defendant. To see this, suppose otherwise. Then there would have to be a rule applicable to the new situation and favoring a decision for the plaintiff that satisfies both clauses of Definition 14—that is, a rule that is both consistent with the background case base and acceptable. But there is no such rule, which we can verify as follows. Beginning with the dimension d_1, imagine the acceptable rules favoring the plaintiff ordered in accord with the salient values along this dimension on which these rules are based. The sequence would begin with the rules

$$\{M^{\pi}_{d_1,12}\} \rightarrow \pi,$$
$$\{M^{\pi}_{d_1,24}\} \rightarrow \pi,$$
$$\{M^{\pi}_{d_1,36}\} \rightarrow \pi,$$
$$\{M^{\pi}_{d_1,48}\} \rightarrow \pi,$$

and then continue indefinitely. It is easy to see, however, that the first two rules in this sequence do not apply to the situation X_{11}, while the others apply both to X_{11} and to the fact situation from c_4, so that a decision on the basis of these rules would be inconsistent. There is therefore no rule addressing the dimension d_1 that favors the plaintiff and is both consistent and applicable. Next, turning to the dimension d_2, the four acceptable rules favoring the plaintiff can be ordered in accord with salient values on which they are based as follows:

$$\{M^{\pi}_{d_2,25}\} \rightarrow \pi,$$
$$\{M^{\pi}_{d_2,50}\} \rightarrow \pi,$$
$$\{M^{\pi}_{d_2,75}\} \rightarrow \pi,$$
$$\{M^{\pi}_{d_2,100}\} \rightarrow \pi.$$

Once more, it is easy to see that the first two of these rules do not apply to the situation X_{11}, while the last two apply both to X_{11} and to the fact situation from c_4, so that a decision on the basis of these rules would be inconsistent. As a result, along this dimension too, there is no rule favoring the plaintiff that is both consistent and acceptable.

It is important to emphasize that the appeal to salient values is not just a formal trick for differentiating the reason and result models of constraint, but that it carries some intuitive weight as well. Consider again the current example: an individual living abroad for thirty months had previously been judged to have changed fiscal domicile on the grounds that she had spent at least a year abroad, and now a new case arises in which an individual has spent twenty nine months abroad. According to the initial version of the reason model, the court could consistently rule that this period abroad does not warrant a change of fiscal domicile on the grounds that it is, say, shorter than twenty nine months and one day. I think that we would all find such a judgment to be objectionable, not on the grounds that it is formally inconsistent with the background case base—by Observation 3, not inconsistent—but on the grounds that, because it is based on such a peculiar value along the dimensional scale, the rule set out by the court appears to be contrived, or crafted simply to justify a desired outcome in a particular case, rather than reflecting a coherent principle.

5 Conclusion

My goal in this paper has been to show—in response to concerns raised by Bench-Capon—how two simple models of precedential constraint, the result and reason models, can be broadened from the standard setting to the dimensional setting, allowing a richer representation of legal information, not just through sets of standard factors, but through dimensions that can take on various values. The path followed in this paper was straightforward, but it led to a surprise. The surprise is that the two models of constraint, which are distinct in the standard setting, collapse together in the richer dimensional setting.

As a response to this collapse, in order to restore a contrast between the result and reason models of constraint, we explored two ways of refining the reason model by requiring that the rules supporting decisions should be subject to additional requirements, beyond simple consistency with the background case base. The first refinement was based on the idea that an earlier rule can be overridden, not by any consistent later rule whatsoever, but only by a consistent later rule that address dimensions different from those addressed by the earlier rule. The second refinement was based on the idea that a later, overriding rule must be, not just consistent, but also acceptable. We concentrated in this paper on a simple interpretation of this idea, according to which the acceptable rules are those based on salient values along a dimensional scale, but of course, the question of whether a rule is acceptable or not could be much more complex than that, involving higher-order considerations of intent or coherence, for example.

In a way, it should come as no surprise that we find ourself focusing on features such as these. Some of the most interesting work in artificial intelligence and law has centered around the evaluation of rules with respect to various higher-order considerations, most particularly the values advanced by those rules—this theme, first sounded in a seminal paper by Donald Berman and Carole Hafner (1993), was later recapitulated by Bench-Capon himself (2002), and developed in the work of, among others, Bench-Capon and Sartor (2003) and Prakken (2002). But what is surprising, at least to me, is that, once we move to the dimensional setting, we should be driven to consider higher-order features of rules simply in order to maintain a distinction between the result model and the reason model of constraint.

Acknowledgements This paper is a light revision and expansion of an earlier version (Horty 2017) that was presented at ICAIL 2017, where it won the Carole Hafner Best Paper Award. This recognition was especially meaningful to me since I have fond memories of working with Carole to organize ICAIL 1995. It was in helping Carole organize that conference that I first learned about the field of AI and Law, and how interesting it could be. I am deeply indebted to Trevor Bench-Capon for raising the issue that motivated this paper in the first place, for his enthusiasm, and for his generous comments on earlier drafts. Apart from providing some additional examples and explanations, along with formal proofs of observations, this revision differs from the earlier version appearing in ICAIL 2017 in only two ways. First, the method of interpreting standard information into dimensional information described here, in the Appendix, differs from the interpretation described in Sect. 4 of that earlier version—both interpretations work, and and they provide different insights. Second, this version of the paper includes a discussion of the first refinement of the reason model, found here in Definition 13, which was mentioned in my ICAIL 2017 presentation but not included in the earlier version for reasons of space. Since my intention in this paper is simply to present in a more careful and systematic way the material from the earlier ICAIL 2017 version, I have not considered work that has appeared since then, including Rigoni (2018) and Bench-Capon and Atkinson (2017). I hope to be able to discuss this more recent work in the future.

Appendices

Interpreting standard information

The body of this paper explores the notion of precedential constraint in two general settings, standard and dimensional. In this first appendix, we show how information from the standard setting can be interpreted in the dimensional setting.

The interpretation described here is based on an idea introduced by Bench-Capon (1999), who represents the overall set of factors through two partial orders, each containing as elements subsets of the factors favoring one side or the other, π or δ; in an anticipation of the reason model, Bench-Capon then sees precedent cases as establishing further ordering relations between elements of these separate partial orders.[9] Here, based on Bench-Capon's representation, we postulate the two dimensions d_π and d_δ, each taking as values subsets of the standard factors favoring the appropriate side. More exactly, where s is a side, the possible values of the dimension d_s are the subsets of F^s, the set of standard factors favoring that side, with these values ordered through the subset relation: if X and Y are subsets of F^s, then

$$X \leq^s Y \text{ if and only if } X \subseteq Y.$$

The idea, of course, is that subsets of F^s containing more standard factors favoring s are values along the dimension d_s that favor the side s more strongly.

Against this background, we now define a *dimensionalization* function $\mathcal{D}$, mapping items from the standard setting into their dimensional counterparts, in five steps. First, where X is a standard fact situation, we take

$$\mathcal{D}(X) = \{\langle d_\pi, X^\pi \rangle, \langle d_\delta, X^\delta \rangle\},$$

[9] In a preliminary version of this paper (Horty 2017, Section 4), I described a different interpretation of standard into dimensional information, with each standard factor thought of as its own dimension, taking the boolean values of 1 or 0 to indicate presence or absence.

where, as we recall, $X^s = X \cap F^s$. $\mathcal{D}(X)$ is thus the dimensional fact situation that assigns to each dimension d_s the set X^s containing those standard factors from X that favor the side s.

There is a slight wrinkle when it comes to interpreting standard reasons in the dimensional setting, since standard reasons are objects of the same type as standard fact situations—sets of standard factors—and so would likewise be mapped by $\mathcal{D}$ into dimensional fact situations, rather than magnitude reasons. As our second step, we therefore introduce an auxiliary function $\mathcal{D}'$ mapping standard reasons into their dimensional counterparts in such a way that, where W is a standard reason favoring the side s, its dimensionalization is

$$\mathcal{D}'(W) = \{M^s_{d_s,W}\}.$$

On the basis of this definition, we can note that, the standard fact situation X satisfies the standard reason W just in case the dimensionalization $\mathcal{D}(X)$ of this situation satisfies $\mathcal{D}'(W)$. This claim can be verified by observing that $\mathcal{D}(X) \models \mathcal{D}'(W)$ holds, by Definition 11, just in case (1) $W \leq^s \mathcal{D}(X)(d_s)$, which is equivalent, since $\mathcal{D}(X)(d_s)$ is X^s, to (2) $W \leq^s X^s$, which is equivalent by the current ordering relation on dimension values to (3) $W \subseteq X^s$, which is equivalent to $X \models W$ by Definitions 1 and 2.

As the third step, where r is a standard rule supporting the outcome s, its dimensionalization $\mathcal{D}(r)$ is defined as a magnitude rule of the form

$$\mathcal{D}'(Premise(r)) \rightarrow s,$$

supporting the same outcome as the original, and taking as its premise the dimensionalization of the standard reason that forms the premise of the original rule. Fourth, where $c = \langle X, r, s \rangle$ is a standard case, its dimensionalization

$$\mathcal{D}(c) = \langle \mathcal{D}(X), \mathcal{D}(r), s \rangle$$

is the dimensional case containing the dimensionalization of the fact situation and rule from the original case, and the same outcome. Fifth, and finally, where Γ is a standard case base, its dimensionalization is

$$\mathcal{D}(\Gamma) = \{\mathcal{D}(c) : c \in \Gamma\}$$

containing dimensionalizations of each case belonging to the original.

We can see these definitions at work by calculating the dimensionalization of the case base $\Gamma_1 = \{c_1\}$, considered earlier, containing the single case $c_1 = \langle X_1, r_1, s_1 \rangle$, where $X_1 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$, where r_1 is $\{f_1^\pi\} \rightarrow \pi$, and where s_1 is π. Descending through the steps in our definition, we have $\mathcal{D}(\Gamma_1) = \{\mathcal{D}(c_1)\}$, with $\mathcal{D}(c_1) = \langle \mathcal{D}(X_1), \mathcal{D}(r_1), s_1 \rangle$, where $\mathcal{D}(X_1) = \{\langle d_\pi, \{f_1^\pi, f_2^\pi\}\rangle, \langle d_\delta, \{f_1^\delta, f_2^\delta\}\rangle\}$ and where $\mathcal{D}(r_1) = \{M^\pi_{d_\pi, \{f_1^\pi\}}\} \rightarrow \pi$.

The question now arises: to what extent are the constraint relations defined in the standard setting preserved under the mapping described here from standard to dimensional information? Or more exactly: given a standard case base Γ and fact situation X, is it the case that a decision for the side s is required in the situation X just in case, moving to the dimensional setting and working against the background of the dimensional case base $\mathcal{D}(\Gamma)$, a decision for s is also required in the dimensional fact situation $\mathcal{D}(X)$? Since we are working with two models of constraint, result and reason, we need to ask the question separately for each model.

Beginning with the result model, it turns out that, here, the notion of constraint defined in the standard setting carries over without change to the dimensional setting.

Observation 4 Let Γ be a standard case base and X a standard fact situation confronting the court. Then the result model of constraint requires a decision for the side s in the situation X if and only if, moving to the dimensional setting and working against the background of the dimensional case base $\mathcal{D}(\Gamma)$, the result model of constraint requires a decision for s in the dimensional fact situation $\mathcal{D}(X)$.

Things are different when we turn to the reason model: here, the concept of constraint defined in the standard setting fails to survive interpretation into the dimensional setting. We can see this by reconsidering our earlier example, introduced in Sect. 2 to illustrate the reason model, in which a court faces the standard fact situation $X_3 = \{f_1^\pi, f_1^\delta\}$ against the background of the standard case base $\Gamma_1 = \{c_1\}$. There, we noted that the reason model of constraint requires the court to decide this situation for the plaintiff, since a decision for the defendant would prioritize $\{f_1^\delta\}$ over $\{f_1^\pi\}$, but the opposite priority ordering is already supported by the background case base. If the example is interpreted in the dimensional setting, however—that is, supposing the court faces the dimensional fact situation $\mathcal{D}(X_3) = \{\langle d_\pi, \{f_1^\pi\}\rangle, \langle d_\delta, \{f_1^\delta\}\rangle\}$ against the background of the dimensional case base $\mathcal{D}(\Gamma_1)$—then the reason model no longer requires a decision for the plaintiff. This fact follows at once from Observation 3, according to which, in the dimensional setting, the reason model requires a decision for a particular side only if the result model also requires a decision for that side. But the result model does not require a decision for the plaintiff in this case, since we do not have $\mathcal{D}(X_1) \leq^\pi \mathcal{D}(X_3)$. Why not? Because there is at least one dimension, namely d_π, whose value in the fact situation $\mathcal{D}(X_3)$ does not favor the plaintiff as strongly as its value in $\mathcal{D}(X_1)$; more precisely, $\mathcal{D}(X_1)(d_\pi)$—that is, the value assigned to d_π in the situation $\mathcal{D}(X_1)$—is $\{f_1^\pi, f_2^\pi\}$ while $\mathcal{D}(X_3)(d_\pi)$ is $\{f_1^\pi\}$, and since $\{f_1^\pi, f_2^\pi\} \subseteq \{f_1^\pi\}$ fails, our ordering on dimension values entails that we do not have $\mathcal{D}(X_1)(d_\pi) \leq^\pi \mathcal{D}(X_3)(d_\pi)$.

If the reason model no longer requires a decision for the plaintiff in the dimensional situation $\mathcal{D}(X_3)$, against the background of the dimensional case base $\mathcal{D}(\Gamma_1)$, there must be some rule on the basis of which this situation can consistently be decided for the defendant. What is this rule? Well, as we saw in our discussion of Example 5, any respect in which a situation is weaker for a side than a situation from a background case already settled for that side can consistently be used as the basis of a rule supporting a decision for the opposite side. In the current example, the situation $\mathcal{D}(X_3)$ is weaker for the plaintiff along the dimension d_π than the situation $\mathcal{D}(X_1)$ from the background case, by taking the value $\{f_1^\pi\}$ rather than $\{f_1^\pi, f_2^\pi\}$, and so can be decided for the defendant on that basis. More precisely, $\mathcal{D}(X_3)$ can be decided for the defendant on the basis of the factor $M^\delta_{d_\pi,\{f_1^\pi\}}$—according to which the value of this situation along the dimension d_π favors the defendant at least as strongly as $\{f_1^\pi\}$. The resulting decision would be represented by the dimensional case $c_{11} = \langle X_{12}, r_{11}, s_{11}\rangle$, where $X_{11} = \mathcal{D}(X_3)$, where $r_{11} = \{M^\delta_{d_\pi,\ \{f_1^\pi\}}\} \rightarrow \delta$, and where s_{11} is δ. The reader can verify that the expanded case base $\Gamma_1 \cup \{c_{11}\}$ is consistent, so that this decision is allowed by the reason model.

It is now worth asking: Once standard information is interpreted into the dimensional setting, can the reason model be modified so that it allows, in the dimensional setting, a pattern of constraint that aligns with that of the standard reason model? In fact, it can—each of the two refinements of the reason model described earlier leads to a pattern of constraint in the dimensional setting matching that from the standard setting.

According to the first refinement of the reason model, set out in Definition 13, a court is required to base its decision in a new situation on a rule that is not only consistent with the existing case base but also separated from any existing rule that applies to the new situation and supports the opposite outcome. This refinement, it turns out, leads to a reason model of constraint in the dimensional setting matching that from the standard setting.

Observation 5 Let Γ be a standard case base and X a standard fact situation confronting the court. Then the reason model of constraint requires a decision for the side s in the situation X if and only if, moving to the dimensional setting and working against the background of the dimensional case base $\mathcal{D}(\Gamma)$, a decision for s in $\mathcal{D}(X)$ is also required by the reason model with the constraint on rule selection subject to the first refinement.

And the general point can be illustrated with our previous example, where, as we can see, the requirement of separation blocks appeal to the new rule $r_{11} = \{M^\delta_{d_\pi,\ \{f_1^\pi\}}\} \rightarrow \delta$ since it is not separated from the existing $\mathcal{D}(r_1) = \{M^\pi_{d_\pi,\ \{f_1^\pi\}}\} \rightarrow \pi$—both of these rule address the dimension d_π.

According to the second refinement of the reason model, set out in Definition 14, a court is required to base its decision on a rule that is not only consistent with the background case base but also, as we said, acceptable—where the notion of acceptability is schematic and can be interpreted in various ways. Earlier, we explored an interpretation of the acceptable rules as those based on salient values. In the present context, where standard information is interpreted in the dimensional setting, let us now suppose that the acceptable rules are the dimensionalizations of standard rules—that is, let us suppose that a dimensional rule r is acceptable if there is some standard rule r' such that r is $\mathcal{D}(r')$. If we think of the acceptable rules in this way, then the second refinement also leads to a reason model of constraint in the dimensional setting matching that from the standard setting.

Observation 6 Let Γ be a standard case base and X a standard fact situation confronting the court. Then the reason model of constraint requires a decision for the side s in the situation X if and only if, moving to the dimensional setting and working against the background of the dimensional case base $\mathcal{D}(\Gamma)$, a decision for s in

$\mathcal{D}(X)$ is also required by the reason model with the constraint on rule selection subject to the second refinement.

And again the general point can be illustrated with our previous example, which depends on the new rule $r_{11} = \{M^{\delta}_{d_\pi, \{f_1^\pi\}}\} \rightarrow \delta$. Although this new rule—suggesting that we should decide for the defendant because the set of factors favoring the plaintiff is not any stronger than $\{f_1^\pi\}$—is a perfectly legitimate rule in the dimensional setting, it is not the dimensionalization of any standard rule, since, in the standard setting, a set of factors favoring the plaintiff, no matter how weak, can support only the plaintiff, not the defendant.

Observations and proofs

Observation 1 Let Γ be a case base with $<_\Gamma$ its derived priority relation. Then Γ is inconsistent if and only if there are cases $c = \langle X, r, s\rangle$ and $c' = \langle Y, r', \bar{s}\rangle$ belonging to Γ such that $Premise(r') <_c Premise(r)$ and $Premise(r) <_{c'} Premise(r')$.

Proof Suppose Γ is inconsistent. Then there are cases $c = \langle X, r, s\rangle$ and $c' = \langle Y, r', \bar{s}\rangle$ belonging to Γ such that $A <_c B$ and $B <_{c'} A$ for some reasons A and B. Since $A <_c B$, we have (1) $X \models A$ and (2) $B \Vdash Premise(r)$. Since $B <_{c'} A$, we have (3) $Y \models B$ and (4) $A \Vdash Premise(r')$. From (1) and (4) we have $X \models Premise(r')$, and of course $Premise(r) \Vdash Premise(r)$, so that $Premise(r') <_c Premise(r)$. From (2) and (3) we have $Y \models Premise(r)$, and of course $Premise(r') \Vdash Premise(r')$, so that $Premise(r) <_{c'} Premise(r')$. □

Observation 2 Let Γ be a consistent case base and X a new fact situation confronting the court, and suppose the result model of constraint requires a decision for the side s in the situation X. Then the reason model of constraint likewise requires a decision for s in this situation.

Proof This result has already been established in the standard setting, using slightly different terminology, as Observation 5 from Horty (2011), and so it is shown here only for the dimensional setting.

Consider, then, a dimensional case base Γ and fact situation X, where the result model of constraint requires a decision for s. Then there is some case $c = \langle Y, r, s\rangle$ from Γ such that $Y \leq^s X$, which means, in the dimensional setting, that $Y(d) \leq^s X(d)$ for each dimension d. Now suppose that the result model does not require a decision for s in the situation X. Then it must be possible to consistently decide X for $\bar{s}$ —that is, there must be some rule r' favoring $\bar{s}$ such that $\Gamma \cup \{c'\}$ is consistent where $c' = \langle X, r', \bar{s}\rangle$.

We can verify that $Y \models Premise(r')$ by showing that Y satisfies each magnitude factor from $Premise(r')$, as follows. Suppose $M^{\bar{s}}_{d,p}$ is a magnitude factor from $Premise(r')$. Then since c' is a case, we have $X \models M^{s}_{d,p}$, that is, $p \leq^{\bar{s}} X(d)$. By assumption, we have $Y(d) \leq^s X(d)$, which yields $X(d) \leq^{\bar{s}} Y(d)$ by duality of the ordering relation on dimension values. By transitivity, this and the previous inequality then tell us that $p \leq^{\bar{s}} Y(d)$, or that $Y \models M^{\bar{s}}_{d,p}$. In the same way, we can verify that $X \models Premise(r)$ by showing that X satisfies each magnitude factor from $Premise(r)$. Suppose $M^{s}_{d,p}$ is a magnitude factor from $Premise(r)$. Then since c is a case, we have $Y \models M^{s}_{d,p}$, or $p \leq^s Y(d)$. We again have $Y(d) \leq^s X(d)$ by assumption, and then $p \leq^s X(d)$ by transitivity, so that $X \models M^{\bar{s}}_{d,p}$.

Since $Y \models Premise(r')$, and of course $Premise(r) \Vdash Premise(r)$, we have $Premise(r') <_c Premise(r)$. And since $X \models Premise(r)$, and of course $Premise(r') \Vdash Premise(r')$, we have $Premise(r) <_{c'} Premise(r')$. But together, these two conclusions tell us that $\Gamma \cup \{c'\}$ is inconsistent, contrary to assumption. □

Observation 3 Let Γ be a consistent dimensional case base and X a new dimensional fact situation confronting the court, and suppose the reason model of constraint requires a decision for the side s in the situation X. Then the result model of constraint likewise requires a decision for the side s in this situation.

Proof We reason by contraposition. Consider a dimensional case base Γ and fact situation X, where the result model of constraint does not require a decision for s. Then there is no case c in Γ such that $Outcome(c) = s$ and $Facts(c) \leq^s X$. In other words, for every c from Γ with $Outcome(c) = s$, it is not the case that $Facts(c)(d) \leq^s X(d)$ for each dimension d—that is, for each such case c, there is some dimension d for which $Facts(c)(d) \leq^s X(d)$ fails. We show that the reason model cannot require a decision for s in the situation X either, by constructing a rule r favoring $\bar{s}$ such that $\Gamma \cup \{c\}$ is consistent where $c = \langle X, r, \bar{s} \rangle$.

If there are no cases in Γ that have been decided for s, then r can be any rule at all favoring $\bar{s}$ whose premise is satisfied by X, so we focus on the more interesting situation in which there are, in fact, cases c from Γ such that $Outcome(c) = s$. For each such case c, let us define d_c as a representative dimension for which $Facts(c)(d) \leq^s X(d)$. (It follows from the argument in the previous paragraph that there is at least one such dimension; if there are more than one, d_c can be chosen arbitrarily.) We know, therefore, that (*) $Facts(c)(d_c) \leq^s X(d_c)$ fails for each c from Γ such that $Outcome(c) = s$.

Now consider the magnitude factor

$$M^{\bar{s}}_{d_c,\, X(d_c)},$$

which holds in any situation in which the value of that situation along the dimension d_c favors the side $\bar{s}$ at least as strongly as $X(d_c)$—that is, at least as strongly as the value of the situation X along the dimension d_c. We form the rule r by collecting together all the factors of this form for each case c from Γ such that $outcome(c) = s$. More precisely, we take r as the rule

$$\{M^{\bar{s}}_{d_c,\, X(d_c)} \ : \ c \in \Gamma \text{ and } Outcome(c) = s\} \rightarrow \bar{s}.$$

In order to establish that $c = \langle X, r, \overline{s} \rangle$ is a case, we verify that $X \models Premise(r)$ by showing that X satisfies each magnitude factor from $Premise(r)$. But this is trivial, since $X \models M^{\overline{s}}_{d_c,\, X(d_c)}$ just in case $X(d_c) \leq^{\overline{s}} X(d_c)$, which is an instance of the reflexivity property of the value ordering.

Next, we establish that $\Gamma \cup \{c\}$ is consistent. Suppose otherwise. In that case, Observation 1 tells us that there is some $c' = \langle Y, r', s \rangle$ belonging to Γ such that $Premise(r') <_c Premise(r)$ and $Premise(r) <_{c'} Premise(r')$. But $Premise(r) <_{c'} Premise(r')$ requires that $Y \models Premise(r)$, which is impossible. Why? Because, since c' is a case from Γ with $Outcome(c') = s$, we know that $Premise(r)$ contains a magnitude factor of the form $M^{\overline{s}}_{d_{c'},\, X(d_{c'})}$, which Y would have to satisfy. But $Y \models M^{\overline{s}}_{d_{c'},\, X(d_{c'})}$ just in case $X(d_{c'}) \leq^{\overline{s}} Y(d_{c'})$, which is equivalent by duality of the value ordering to $Y(d_{c'}) \leq^{s} X(d_{c'})$, which is equivalent, since $Facts(c') = Y$, to $Facts(c')(d_{c'}) \leq^{s} X(d_{c'})$, which we know to be false by (*) above. □

Observation 4 Let Γ be a standard case base and X a standard fact situation confronting the court. Then the result model of constraint requires a decision for the side s in the situation X if and only if, moving to the dimensional setting and working against the background of the dimensional case base $\mathcal{D}(\Gamma)$, the result model of constraint requires a decision for s in the dimensional fact situation $\mathcal{D}(X)$.

Proof We begin by verifying that, if X and Y are standard fact situations, then (1) $X \leq^{s} Y$ holds in the standard setting just in case, moving to the dimensional setting, we have (2) $\mathcal{D}(X) \leq^{s} \mathcal{D}(Y)$. In the standard setting, (1) is equivalent to (3) $X^{s} \subseteq Y^{s}$ and (4) $Y^{\overline{s}} \subseteq X^{\overline{s}}$, while in the dimensional setting, (2) means that $\mathcal{D}(X)(d) \leq^{s} \mathcal{D}(Y)(d)$ for each dimension d. But based on our interpretation of standard information into the dimensional setting, there are only two dimensions to consider, d_s and $d_{\overline{s}}$, so that (2) is equivalent to (5) $\mathcal{D}(X)(d_s) \leq^{s} \mathcal{D}(Y)(d_s)$ and (6) $\mathcal{D}(X)(d_{\overline{s}}) \leq^{s} \mathcal{D}(Y)(d_{\overline{s}})$. Since, according to our interpretation, $\mathcal{D}(X)(d_s)$ is X^{s} and $\mathcal{D}(Y)(d_s)$ is Y^{s}, (5) is just the statement (7) $X^{s} \leq^{s} Y^{s}$, where X^{s} and Y^{s} are to be interpreted as values along the dimension d_s, which holds, according to our dimensional value ordering, just in case $X^{s} \subseteq Y^{s}$, which is simply (3). And since, according to our interpretation, $\mathcal{D}(X)(d_{\overline{s}})$ is $X^{\overline{s}}$ and $\mathcal{D}(Y)(d_{\overline{s}})$ is $Y^{\overline{s}}$, (6) is just the statement (8) $X^{\overline{s}} \leq^{s} Y^{\overline{s}}$, where $X^{\overline{s}}$ and $Y^{\overline{s}}$ are to be interpreted as values along the dimension $d_{\overline{s}}$. By duality of the ordering relation on dimension values, (8) is equivalent to (9) $Y^{\overline{s}} \leq^{\overline{s}} X^{\overline{s}}$, which holds, according to our dimensional value ordering, just in case $Y^{\overline{s}} \subseteq X^{\overline{s}}$, which is simply (4).

Next, we turn to the result itself. Reasoning from left to right (the other direction is similar), with Γ a standard case base and X a standard fact situation, suppose the result model requires a decision for s in X. Then there is a case c in Γ with $Outcome(c) = s$ such that $Facts(c) \leq^{s} X$. But by our definition of the dimensionalization function, the case $\mathcal{D}(c)$ belongs to $\mathcal{D}(\Gamma)$, we still have $Outcome(\mathcal{D}(c)) = s$, and as we have just seen, $\mathcal{D}(Facts(c)) \leq^{s} \mathcal{D}(X)$, so that the result model requires a decision for s in the dimensional setting as well. □

Lemma 1 *Let X be a standard fact situation and W a standard reason. Then $X \models W$ if and only if $\mathcal{D}(X) \models \mathcal{D}'(W)$.*

Proof This fact is verified in Section 5.1 of the text, immediately after the definition of the $\mathcal{D}'$ function. □

Lemma 2 *Let Γ be a standard case base and $c = \langle X, r, s\rangle$ and $c' = \langle Y, r', \overline{s}\rangle$ cases belonging to Γ. Then $Premise(r') <_c Premise(r)$ if and only if $\mathcal{D}'(Premise(r)) <_{\mathcal{D}(c)} \mathcal{D}'(Premise(r))$.*

Proof $Premise(r') <_c Premise(r)$ is equivalent to (1) $X \models Premise(r')$ and (2) $Premise(r) \Vdash Premise(r)$, while $\mathcal{D}'(Premise(r')) <_{\mathcal{D}(c)} \mathcal{D}'(Premise(r))$ is equivalent to (3) $\mathcal{D}(X) \models \mathcal{D}(Premise(r'))$ and (4) $\mathcal{D}'(Premise(r)) \Vdash \mathcal{D}'(Premise(r))$. But (2) and (4) are obvious and (1) is equivalent to (3) by Lemma 1. Therefore $Premise(r') <_c Premise(r)$ is equivalent to $\mathcal{D}'(Premise(r')) <_{\mathcal{D}(c)} \mathcal{D}'(Premise(r))$. □

Lemma 3 *Let Γ be a consistent standard case base, X a standard fact situation, and r a standard rule supporting the outcome s. Then if $\mathcal{D}(\Gamma) \cup \{\langle \mathcal{D}(X), \mathcal{D}(r), s\rangle\}$ is consistent, so is $\Gamma \cup \{\langle X, r, s\rangle\}$.*

Proof Suppose for contraposition that $\Gamma \cup \{c\}$ is inconsistent where $c = \langle X, r, s\rangle$. Then Γ must contain some case $c' = \langle Y, r', \overline{s}\rangle\}$ allowing us to show that $Premise(r') <_c Premise(r)$ and $Premise(r) <_{c'} Premise(r')$. But then by Lemma 2 we will also have $\mathcal{D}'(Premise(r')) <_{\mathcal{D}(c)} \mathcal{D}'(Premise(r))$ and $\mathcal{D}'(Premise(r)) <_{\mathcal{D}(c')} \mathcal{D}'(Premise(r'))$, and since $\mathcal{D}(c')$ must belong to $\mathcal{D}(\Gamma)$, it follows that $\mathcal{D}(\Gamma) \cup \{\mathcal{D}(c)\}$ is inconsistent as well. □

Observation 5 Let Γ be a standard case base and X a standard fact situation confronting the court. Then the reason model of constraint requires a decision for the side s in the situation X if and only if, moving to the dimensional setting and working against the background of the dimensional case base $\mathcal{D}(\Gamma)$, a decision for s in $\mathcal{D}(X)$ is also required by the reason model with the constraint on rule selection subject to the first refinement.

Proof Since the two directions are similar, we prove only the left to right direction. Assume, then, that the reason model requires a decision for s in the situation X. Then there must be some standard rule r supporting s such that $\Gamma \cup \{\langle X, r, s\rangle\}$ is consistent, and there can be no standard rule consistently supporting the opposite side—that is, no r' supporting $\overline{s}$ such that $\Gamma \cup \{\langle X, r', \overline{s}\rangle\}$ is consistent.

Now suppose that, moving to the dimensional setting, the reason model subject to the first refinement does not require a decision for s in the situation $\mathcal{D}(X)$. Then there must be some dimensional rule r'' supporting $\overline{s}$ such that $\mathcal{D}(\Gamma) \cup \{\langle \mathcal{D}(X), r'', \overline{s}\rangle\}$ is consistent. Since $X \models Premise(r)$, we know from Lemma 1 that

$\mathcal{D}(X) \models Premise(\mathcal{D}(r))$, and of course $Outcome(\mathcal{D}(r)) = s$. By the refinement of the reason model, r'' must therefore be separated from $\mathcal{D}(r)$.

Since r is a standard rule, it has the form $W \rightarrow s$ for some $W \subseteq F^s$, with the consequence that $\mathcal{D}(r)$ has the form $\{M^s_{d_s,W}\} \rightarrow s$. The rule $\mathcal{D}(r)$, therefore, addresses the dimension d_s, so that, by separation, r'' must address the dimension $d_{\bar{s}}$. The rule r'' must therefore have the form $\{M^{\bar{s}}_{d_{\bar{s}},V}\} \rightarrow \bar{s}$ for some $V \subseteq F^{\bar{s}}$, so that r'' is $\mathcal{D}(r''')$ where r''' is the standard rule $V \rightarrow \bar{s}$. But since $\mathcal{D}(\Gamma) \cup \{\langle \mathcal{D}(X), r'', \bar{s} \rangle\}$ is consistent, we can conclude from Lemma 3 that $\Gamma \cup \{X, r''', \bar{s}\}$ is consistent as well, contrary to our assumption that the standard model requires a decision for s in X. □

Observation 6 Let Γ be a standard case base and X a standard fact situation confronting the court. Then the reason model of constraint requires a decision for the side s in the situation X if and only if, moving to the dimensional setting and working against the background of the dimensional case base $\mathcal{D}(\Gamma)$, a decision for s in $\mathcal{D}(X)$ is also required by the reason model with the constraint on rule selection subject to the second refinement.

Proof Again we prove only the left to right direction. Assume, then, that the reason model requires a decision for s in the situation X, and suppose that, moving to the dimensional setting, the reason model subject to the second refinement does not require a decision for s in the situation $\mathcal{D}(X)$. Then there must be a dimensional rule r such that (1) $\mathcal{D}(\Gamma) \cup \{\langle \mathcal{D}(X), r, \bar{s} \rangle\}$ is consistent and (2) r is the dimensionalization of some standard rule r'. It then follows from Lemma 3 that $\Gamma \cup \{\langle X, r', \bar{s} \rangle\}$ is consistent as well, contrary to the assumption that the reason model in the standard setting requires a decision for s. □

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