

# Norm Change in the Common Law

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**Abstract** An account of legal change in a common law system is developed. Legal change takes place incrementally through court decisions that are constrained by previous decisions in other courts. The assumption is that a court's decision has to be consistent with the rules set out in earlier court decisions. However, the court is allowed to make add new distinctions and therefore make a different decision based on factors not present in the previous decision. Two formal models of this process are presented. The first model is based on refinement of (the set of factors taken into account in) the set of previous cases on which a decision is based. In the second model the focus is on a preference ordering on reasons. The court is allowed to supplement, but not to revise the preference ordering on reasons that can be inferred from previous cases. The two accounts turn out to be equivalent. A court can make a consistent decision even if the case base is not consistent; the important requirement is that no new inconsistencies should be added to the case base.

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## 1 Introduction

Among David Makinson's many achievements in logic, none is more important than his development, along with Carlos Alchourrón and Peter Gärdenfors, of the AGM theory of belief change.

The origin of that work has now been documented—in David's obituary of Alchourrón, in Gärdenfors's brief history, and in David's own reflections—and it

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is, in many ways, a dramatic saga.<sup>1</sup> From David's perspective, it began with the problem of norm change in the law, or more specifically, with Alchourrón's interest, together with that of his colleague Eugenio Bulygin, in the concept of derogation: the removal of a norm from a system of norms, such as a legal code.<sup>2</sup> The difficulty is that the individual norm to be derogated might not simply be listed in the legal code, but instead, or in addition, implied by other individual norms from the code, or by sets of other norms taken together. In the latter case, it will be possible for the derogation of a particular norm to be achieved in a number of ways, depending on which adjustments are made to the set of norms supporting it; the result is, therefore, indeterminate.

David reports that he did not, at first, see much of interest in the concept of derogation for exactly this reason, the indeterminacy of its result, which he viewed as "just an unfortunate fact of life . . . about which formal logic could say little or nothing". By the end of the 1970s, however, he and Alchourrón had managed to frame the issue in a way that was amenable to formal analysis, and published the outcome in the second of Risto Hilpinen's two influential collections on deontic logic.<sup>3</sup> Just as they were completing this paper, they realized that both the issues under consideration and their logical analysis could be seen in a more general light—as a matter of belief revision in general, not just norm revision. This perspective was adopted in a second paper, submitted to *Theoria*.<sup>4</sup>

As it happens, the editor of that journal was then Peter Gärdenfors, who was working on formally similar problems, though with a distinct philosophical motivation—Gärdenfors had been exploring the semantics of conditionals, not norm change—and a collaboration was joined. Of course, there would have been differences: Gärdenfors's approach had been largely postulational, while the approach of Alchourrón and Makinson was definitional; Alchourrón and Makinson had focused on derogation, now called contraction, as the fundamental operation, with revision defined through the Levi identity, while Gärdenfors took the opposite route, treating revision as fundamental with contraction defined through the Harper identity. Nevertheless, David describes the collaboration as "a dream", with differences resolved and further progress achieved, in those days before email, through a series of longhand letters "circulating incessantly between Buenos Aires, Lund, Beirut, and Paris". The result was the initial AGM paper, which, taken together with subsequent work on the topic by the original authors and many others—in fields including philosophy, computer science, economics, and psychology—stands as one of the great success stories from the past 25 years of philosophical logic.<sup>5</sup>

<sup>1</sup> See Makinson (1996, 2003) and Gärdenfors (2011).

<sup>2</sup> The term "derogation" is often used to refer only to limitation of a norm, while its full removal is described as an "abrogation". My terminology here follows that of Alchourrón and Makinson (1981).

<sup>3</sup> Alchourrón and Makinson (1981).

<sup>4</sup> Alchourrón and Makinson (1982).

<sup>5</sup> Alchourrón, Gärdenfors, and Makinson (1985).



I will not try to advance this story here. Instead, I want to return to its roots: the problem of norm change in the law. It is natural that Alchourrón, as an Argentinian, from a civil law country, would explore this problem in the context of a changing legal canon, an evolving body of rules. But any jurist working in the United Kingdom, America, Canada, Australia, New Zealand, or any of the other common law countries, if asked about norm change in the law, would think first, not about explicit modifications to a legal code, but about the common law itself. And here, the process of norm change is typically more gradual, incremental, and mediated by the common law doctrine of precedent, according to which the decisions of earlier courts generalize to constrain the decisions of later courts, while still allowing these later courts a degree freedom in responding to fresh circumstances.

On what is, perhaps, the standard view, the constraints of precedent are themselves carried through rules: a court facing a particular problem situation either invokes a previous common law rule or articulates a new one to justify its decision in that case, and this rule is then generally thought to determine the decisions that might be reached in any future case to which it applies. There are, however, two qualifications. Some courts, depending on their place in the judicial hierarchy, have the power to *overrule* the decisions of earlier courts. The effect of overruling is much like that of derogation: the normative force of a case that has been overruled is removed entirely. Overruling is, therefore, radical, but it is also rare, and not a form of norm change that I will discuss here.

Although only certain courts have the power to overrule earlier decisions, all courts are thought to have the power of *distinguishing* later cases—the power, that is, to point out important differences between the facts present in some later case and those of earlier cases, and so modifying the rules set out in those earlier cases to avoid what they feel would be an inappropriate application to the later case. Of course, later courts cannot modify the rules set out by earlier courts entirely at will, in any way whatsoever. There must be some restrictions on this power, and the most widely accepted restrictions are those first set out explicitly by Joseph Raz, although, as Raz acknowledges, the account owes much previous work of A. W. B. Simpson.<sup>6</sup> According to this account, any later modification of an earlier rule must satisfy two conditions: first, the modification can consist only in the addition of further qualifications, which will thus narrow the original rule; and second, the modified rule must continue to yield the original outcome in the case in which it was introduced, as well as in any further cases in which this rule was applied.

In recent work, motivated in part by research from the field of Artificial Intelligence and Law, as well as by a previous proposal due to Grant Lamond, I developed an account of precedent in the common law according to which constraint is not a matter of rules at all, but of reasons.<sup>7</sup> More exactly, I suggested that what is important about a precedent case is the previous court's assessment of the balance of reasons

<sup>6</sup> See Raz (1979, pp. 180–209) and Simpson (1961).

<sup>7</sup> See Horty (2011), and then Horty and Bench-Capon (2012) for a development of this account within the context of related research from Artificial Intelligence and Law; see Lamond (2005) for his earlier proposal.



presented by that case; later courts are then constrained, not to follow some rule set out by the earlier court, but to reach a decision that is consistent with the earlier court's assessment of the balance of reasons.

The account I propose is precise and allows, I believe, for a good balance between the constraints imposed by previous decisions and the freedoms granted to later courts for developing the law. But my account is also unusual, especially in abstaining from any appeal to rules in its treatment of the common law, and the question immediately arises: what is the relation between this account of precedential constraint, developed in terms of reasons, and the standard account, relying on rules?

The goal of the current paper is to answer this question. More precisely, what I show is that, even though the account of precedential constraint developed in terms of reasons was introduced as an alternative to the standard account, in terms of rules, it turns out that these two accounts are, in an important sense, equivalent. Establishing this result requires a precise statement of the notion of constraint at work in the standard account, which is offered in Sect. 3 of this paper, after basic concepts are introduced in Sect. 2. The account of precedential constraint in terms of reasons is reviewed in Sects. 4, and 5 establishes its equivalence with the standard account. Section 6 mentions some of the formal issues raised by this work; a discussion of the philosophical motivation is reserved for a companion paper.

## 2 Factors, Rules, and Cases

I follow the work of Edwina Rissland, Kevin Ashley, and their colleagues in supposing that the situation presented to the court in a legal case can usefully be represented as a set of *factors*, where a factor stands for a legally significant fact or pattern of facts.<sup>8</sup> Cases in different areas of the law will be characterized by different sets of factors, of course. In the domain of trade secrets law, for example, where the factor-based analysis has been developed most extensively, a case will typically concern the issue of whether the defendant has gained an unfair competitive advantage over the plaintiff through the misappropriation of a trade secret; and here the factors involved might turn on, say, questions concerning whether the plaintiff took measures to protect the trade secret, whether a confidential relationship existed between the plaintiff and the defendant, whether the information acquired was reverse-engineerable or in some other way publicly available, and the extent to which this information did, in fact, lead to a real competitive advantage for the defendant.<sup>9</sup>

<sup>8</sup> See Rissland and Ashley (1987) and then Ashley (1989, 1990) for an introduction to the model; see also, Rissland (1990) for an overview of research in Artificial Intelligence and Law that places this work in a broader context.

<sup>9</sup> Aleven (1997) has analyzed 147 cases from trade secrets law in terms of a factor hierarchy that includes 5 high-level issues, 11 intermediate-level concerns, and 26 base-level factors. The resulting knowledge base is used in an intelligent tutoring system for teaching elementary skills in legal argumentation, which has achieved results comparable to traditional methods of instruction in controlled studies; see Aleven and Ashley (1997).



We will assume, as usual, that factors have polarities, always favoring one side or another. In the domain of trade secrets law, once again, the presence of security measures favors the plaintiff, since it strengthens the claim that the information secured was a valuable trade secret; reverse-engineerability favors the defendant, since it suggests that the product information might have been acquired through proper means. The paper is based, furthermore, on the simplifying assumption that the reasoning under consideration involves only a single step, proceeding from the factors present in a case immediately to a decision—in favor of the plaintiff or the defendant—rather than moving through a series of intermediate legal concepts.<sup>10</sup>

Formally, then, we will let  $F^\pi = \{f_1^\pi, \dots, f_n^\pi\}$  represent the set of factors favoring the plaintiff and  $F^\delta = \{f_1^\delta, \dots, f_m^\delta\}$  the set of factors favoring the defendant. Since each factor favors one side of the other, we can suppose that the entire set  $F$  of legal factors is exhausted by these two sets:  $F = F^\pi \cup F^\delta$ . A *fact situation*  $X$ , of the sort presented in a legal case, can then be defined as some particular subset of the overall set of factors:  $X \subseteq F$ .

A *precedent case* will be represented as a fact situation together with an outcome as well as a rule through which that outcome is reached. Such a case can be defined as a triple of the form  $c = \langle X, r, s \rangle$ , where  $X$  is a fact situation containing the legal factors present in the case,  $r$  is the rule of the case, and  $s$  is its outcome.<sup>11</sup> We define three functions—*Factors*, *Rule*, and *Outcome*—to map cases into their component parts, so that, in the case  $c$  above, for example, we would have  $Factors(c) = X$ ,  $Rule(c) = r$ , and  $Outcome(c) = s$ .

Given our assumption that reasoning proceeds in a single step, we can suppose that the *outcome*  $s$  of a case is always either a decision in favor of the plaintiff or a decision in favor of the defendant, with these two outcomes represented as  $\pi$  or  $\delta$  respectively; and where  $s$  is a particular outcome, a decision for some side, we suppose that  $\bar{s}$  represents a decision for the opposite side, so that  $\bar{\pi} = \delta$  and  $\bar{\delta} = \pi$ . Where  $X$  is a fact situation, we let  $X^s$  represent the factors from  $X$  that support the side  $s$ ; that is,  $X^\pi = X \cap F^\pi$  and  $X^\delta = X \cap F^\delta$ .

Rules are to be defined in terms of reasons, where a *reason for a side* is a set of factors favoring that side. A *reason* can then be defined as a set of factors favoring one side or another. To illustrate:  $\{f_1^\pi, f_2^\pi\}$  is a reason favoring the side  $\pi$ , and so a reason, while  $\{f_1^\delta\}$  is a reason favoring  $\delta$ , and likewise a reason; but the set  $\{f_1^\pi, f_1^\delta\}$  is not a reason, since the factors it contains do not uniformly favor one side or another.

A statement of the form  $X \models R$  indicates that the fact situation  $X$  *satisfies* the reason  $R$ , or that the reason *holds* in that situation; this idea can be defined by stipulating that

$$X \models R \text{ just in case } R \subseteq X,$$

<sup>10</sup> Both of the assumptions mentioned in this paragraph are discussed in Horty (2011).

<sup>11</sup> For the purpose of this paper, I simplify by assuming that the rule underlying a court's decision is plain, ignoring the extensive literature on methods for determining the rule, or *ratio decidendi*, of a case. I will also assume that a case always contains a single rule, ignoring situations in which a judge might offer several rules for a decision, or in which a court reaches a decision by majority, with different judges offering different rules, or in which a judge might simply render a decision in a case without setting out any general rule at all.

and then extended in the usual way to statements  $\phi$  and  $\psi$  formed by closing the reasons under conjunction and negation:

$$\begin{aligned} X \models \neg\phi & \text{ if and only if it fails that } X \models \phi, \\ X \models \phi \wedge \psi & \text{ if and only if } X \models \phi \text{ and } X \models \psi. \end{aligned}$$

We stipulate, in the usual way, that  $\phi$  implies a statement  $\phi \vdash \psi$ —that is,  $\phi$  implies  $\psi$ —just in case  $X \models \psi$  whenever  $X \models \phi$ .

Given this notion of a reason, a rule can now be defined as a pair whose premise is a certain kind of conjunction of reasons and their negations and whose conclusion is an outcome, a decision favoring one side or the other. More specifically, where  $R^s$  is a single reason for the side  $s$  and  $R_1^{\bar{s}}, \dots, R_i^{\bar{s}}$  are zero or more reasons for the opposite side, then a *rule for the side  $s$*  has the form

$$R^s \wedge \neg R_1^{\bar{s}} \wedge \dots \wedge \neg R_i^{\bar{s}} \rightarrow s$$

and a *rule* is simply a rule for one side or the other; the idea, of course is that, when the reason  $R^s$  favoring  $s$  holds in some situation, and none of the reasons  $R_1^{\bar{s}}, \dots, R_i^{\bar{s}}$  favoring the opposite side hold, then  $r$  requires a decision for the side  $s$ . Given a rule  $r$  of this form, we define functions *Premise*, *Premise<sup>s</sup>*, *Premise <sup>$\bar{s}$</sup>* , and *Conclusion* picking out its premise, the positive part of its premise, the negative part, and its conclusion, all as follows:

$$\begin{aligned} \text{Premise}(r) &= R^s \wedge \neg R_1^{\bar{s}} \wedge \dots \wedge \neg R_i^{\bar{s}}, \\ \text{Premise}^s(r) &= R^s, \\ \text{Premise}^{\bar{s}}(r) &= \neg R_1^{\bar{s}} \wedge \dots \wedge \neg R_i^{\bar{s}}, \\ \text{Conclusion}(r) &= s. \end{aligned}$$

We can then say that  $r$  *applies* in a fact situation  $X$  just in case  $X \models \text{Premise}(r)$ .

Let us return, now, to the concept of a precedent case  $c = \langle X, r, s \rangle$ , containing a fact situation  $X$  along with a rule  $r$  leading to the outcome  $s$ . In order for this concept to make sense, we impose two coherence constraints. The first is that the rule contained in the case must actually apply to the facts of the case, or that  $X \models \text{Premise}(r)$ . The second is that the conclusion of the precedent rule must match the outcome of the case itself, or that  $\text{Conclusion}(r) = \text{Outcome}(c)$ .

These various concepts and constraints can be illustrated through the concrete case  $c_1 = \langle X_1, r_1, s_1 \rangle$ , containing the fact situation  $X_1 = \{f_1^\pi, f_2^\pi, f_3^\pi, f_1^\delta, f_2^\delta, f_3^\delta, f_4^\delta\}$ , with three factors favoring the plaintiff and four favoring the defendant, where  $r_1$  is the rule  $\{f_1^\pi, f_2^\pi\} \wedge \neg\{f_5^\delta\} \wedge \neg\{f_4^\delta, f_6^\delta\} \rightarrow \pi$ , and where the outcome  $s_1$  is  $\pi$ , a decision for the plaintiff. Since we have both  $X_1 \models \text{Premise}(r_1)$  and  $\text{Conclusion}(r_1) = \text{Outcome}(c_1)$ , it is clear that the case satisfies our two coherence constraints: the precedent rule is applicable to the fact situation: and the conclusion of the precedent rule matches the outcome of the case. This particular precedent, then, represents a case in which the court decided for the plaintiff by applying or introducing a rule according to which the presence of the factors  $f_1^\pi$  and  $f_2^\pi$ , together with the absence



of the factor  $f_5^\delta$ , as well as the absence of the pair of factors  $f_4^\delta$  and  $f_6^\delta$ , leads to decision for the plaintiff.

With this notion of a precedent case in hand, we can now define a *case base* as a set  $\Gamma$  of precedent cases. It is a case base of this sort that will be taken to represent the common law in some area, and to constrain the decisions of future courts.

### 3 Constraint by Rules

We now turn to the standard account of precedential constraint, in terms of rules that can be modified. I motivate this account by tracing a three simple examples of legal development according to the standard view, generalizing from these examples, and then characterizing what I take to be the standard notion of precedential constraint in terms of this generalization.

To begin with, then, suppose that the background case base is  $\Gamma_1 = \{c_2\}$ , containing only the single precedent case  $c_2 = \langle X_2, r_2, s_2 \rangle$ , with  $X_2 = \{f_1^\pi, f_1^\delta\}$ , where  $r_2 = \{f_1^\pi\} \rightarrow \pi$ , and where  $s_2 = \pi$ ; this precedent represents a situation in which a prior court, confronted with the conflicting factors  $f_1^\pi$  and  $f_1^\delta$ , decided for  $\pi$  on the basis of  $f_1^\pi$ . Now imagine that, against the background of this case base, a later court is confronted with the new fact situation  $X_3 = \{f_1^\pi, f_2^\delta\}$ , and takes the presence of the new factor  $f_2^\delta$  as sufficient to justify a decision for  $\delta$ . Of course, the previous rule  $r_2$  applies to the new fact situation, apparently requiring a decision for  $\pi$ . But according to the standard account, the court can decide for  $\delta$  all the same by distinguish the new fact situation from that of the case in which  $r_2$  was introduced—pointing out that the new situation, unlike that of the earlier case, contains the factor  $f_2^\delta$ , and so declining to apply the earlier rule on that basis.

The result of this decision, then, is that the original case base is changed in two ways. First, by deciding the new situation for  $\delta$  on the basis of  $f_2^\delta$ , the court supplements this case base with the new case  $c_3 = \langle X_3, r_3, s_3 \rangle$ , where  $X_3$  is as above, where  $r_3 = \{f_2^\delta\} \rightarrow \delta$ , and where  $s_3 = \delta$ . And second, by declining to apply the earlier  $r_2$  to the new situation due to the presence of  $f_2^\delta$ , the court, in effect, modifies this earlier rule so that it now carries the force of  $r_2' = \{f_1^\pi\} \wedge \neg\{f_2^\delta\} \rightarrow \pi$ . The new case base is thus  $\Gamma_1' = \{c_2', c_3\}$ , with  $c_2' = \langle X_2', r_2', s_2' \rangle$  where  $X_2' = X_2$ , where  $r_2'$  is as above, and where  $s_2' = s_2$ , and with  $c_3$  as above.

The process could continue, of course. Suppose now that, against the background of the modified case base  $\Gamma_1' = \{c_2', c_3\}$ , another court is confronted with the further fact situation  $X_4 = \{f_1^\pi, f_3^\delta\}$ , and again takes the new factor  $f_3^\delta$  as sufficient to justify a decision for  $\delta$ , in spite of the fact that even the modified rule  $r_2'$  requires a decision for  $\pi$ . Once again, this decision changes the current case base in two ways: first, supplementing this case base with a new case representing the current decision, and second, further modifying the previous rule to avoid a conflicting result in the current case. The resulting case base is therefore  $\Gamma_1'' = \{c_2'', c_3, c_4\}$ , with  $c_2'' = \langle X_2'', r_2'', s_2'' \rangle$  as a modification of the previous  $c_2'$ , where  $X_2'' = X_2'$ ,

where  $s_2'' = s_2'$ , and now where  $r_2'' = \{f_1^\pi\} \wedge \neg\{f_2^\delta\} \wedge \neg\{f_3^\delta\} \rightarrow \pi$ , with  $c_3$  is as above, and with  $c_4 = \langle X_4, r_4, s_4 \rangle$  representing the current decision, where  $X_4$  is as above, where  $r_4 = \{f_3^\delta\} \rightarrow \delta$ , and where  $s_4 = \delta$ .

As our second example, suppose that the background case base is  $\Gamma_2 = \{c_2, c_5\}$ , with  $c_2$  as above, and with  $c_5 = \langle X_5, r_5, s_5 \rangle$ , where  $X_5 = \{f_1^\pi, f_2^\delta\}$ , where  $r_5 = \{f_1^\pi\} \rightarrow \pi$ , and where  $s_5 = \pi$ . This case base represents a pair of prior decisions for  $\pi$  on the basis of  $f_1^\pi$ , in spite of the conflicting factors  $f_1^\delta$ , in one case, and  $f_2^\delta$ , in the other. Now suppose that, against this background, a later court confronts the new situation  $X_6 = \{f_1^\pi, f_1^\delta, f_2^\delta\}$ , and decides that, although earlier cases favored  $f_1^\pi$  over the conflicting  $f_1^\delta$  and  $f_2^\delta$  presented separately, the combination of  $f_1^\delta$  and  $f_2^\delta$  together justifies a decision for  $\delta$ . Again, this decision supplements the existing case base with the new case  $c_6 = \langle X_6, r_6, s_6 \rangle$ , where  $X_6$  as above, where  $r_6 = \{f_1^\delta, f_2^\delta\} \rightarrow \delta$ , and where  $s_6 = \delta$ . But here, the rules from both of the existing cases,  $c_2$  and  $c_5$ , must be modified to block application to situations in which  $f_1^\delta$  and  $f_2^\delta$  appear together, and so now carry the force of  $r_2' = r_5' = \{f_1^\pi\} \wedge \neg\{f_1^\delta, f_2^\delta\} \rightarrow \pi$ . The case base resulting from this decision is thus  $\Gamma_2' = \{c_2', c_5', c_6, \}$ , with  $c_2' = \langle X_2', r_2', s_2' \rangle$  where  $X_2' = X_2$ , where  $r_2'$  as above, and where  $s_2' = s_2$ , with  $c_5' = \langle X_5', r_5', s_5' \rangle$  where  $X_5' = X_5$ , where  $r_5'$  is as above, and where  $s_5' = s_5$ , and with  $c_6$  as above.

Finally, suppose the background case base is  $\Gamma_3 = \{c_2, c_7\}$  again with  $c_2$  as above, but with  $c_7 = \langle X_7, r_7, s_7 \rangle$ , where  $X_7 = \{f_2^\pi, f_2^\delta\}$ , where  $r_7 = \{f_2^\pi\} \rightarrow \pi$ , and where  $s_7 = \pi$ . This case base represents a pair of previous decisions for  $\pi$ , one on the basis of  $f_1^\pi$  in spite of the conflicting  $f_1^\delta$ , and one on the basis of  $f_2^\pi$  in spite of the conflicting  $f_2^\delta$ . Now imagine that a later court confronts the new situation  $X_8 = \{f_1^\pi, f_2^\delta\}$ , containing two factors that have not yet been compared, and concludes that  $f_2^\delta$  is sufficient to justify a decision for  $\delta$ , in spite of the conflicting  $f_1^\pi$ . Once again, the earlier rule  $r_2$  must be taken to have the force of  $r_2' = \{f_1^\pi\} \wedge \neg\{f_2^\delta\} \rightarrow \pi$ , in order not to conflict with the current decision. In this case, however, the new rule cannot be formulated simply as  $\{f_2^\delta\} \rightarrow \delta$ , but must now take the form of  $r_8 = \{f_2^\delta\} \wedge \neg\{f_2^\pi\} \rightarrow \delta$ , in order not to conflict with the decision for  $\pi$  previously reached in  $c_7$ . This scenario, then, is one in which modifications are forced in both directions: a previous rule must be modified to avoid conflict with the current decision, while at the same time, the rule of the current case must be hedged to avoid conflict with a previous decision. The resulting case base is  $\Gamma_3' = \{c_2', c_7, c_8, \}$ , with  $c_2' = \langle X_2', r_2', s_2' \rangle$ , where  $X_2' = X_2$ , where  $r_2'$  is as above, and where  $s_2' = s_2$ , with  $c_7$  as above, and with  $c_8 = \langle X_8, r_8, s_8 \rangle$ , where  $X_8$  as above, where  $r_8$  as above, and where  $s_8 = \delta$ .

Each of these examples describes a scenario in which a sequence of fact situations are confronted, decisions are reached, rules are formulated to justify the decisions, and rules are modified to accommodate later, or earlier, decisions. It is interesting, and somewhat surprising, to note that, as long as all decisions can be accommodated, with rules properly modified to avoid conflicts, then the order in which cases are confronted is irrelevant. To put this point precisely, let us stipulate that, where  $c = \langle X, r, s \rangle$  is a precedent case decided for the side  $s$ , the *reason* for this decision is *Premise*<sup>s</sup>( $r$ ), the positive part of the premise of the case rule; and suppose that a case base has been constructed through the process of considering fact situations in some particular



sequence, in each case rendering a decision for some particular reason and modifying other rules accordingly. It then turns out that, as long as the same decisions are rendered for the same reasons, the same case base will be constructed, with all rules modified in the same way, regardless of the sequence in which the fact situations are considered. Indeed, the fact situations need not be considered in any sequence at all: as long as the set of decisions in these situations is capable of being accommodated through appropriate rule modifications, then all the rules can be modified at once, through a process of case base *refinement*.

This process of transforming a case base  $\Gamma$  into its refinement can be described informally as follows: first, for each case  $c$  belonging to  $\Gamma$ , decided for some side and for some particular reason, collect together into  $\Gamma_c$  all of the cases in which that reason hold, but which were decided for the other side; next, for each such case  $c'$  from  $\Gamma_c$ , take the negation of the reason for which that case was decided, and then conjoin all of these negated reasons together; finally, replace the rule from the original case  $c$  with the new rule that results when this complex conjunction is itself conjoined with the reason for the original decision. And this informal description can be transformed at once into a formal definition.

**Definition 1 (Refinement of a case base)** Where  $\Gamma$  is a case base, its refinement—written,  $\Gamma^+$ —is the set that results from carrying out the following procedure. For each case  $c = \langle X, r, s \rangle$  belonging to  $\Gamma$ :

1. Let

$$\Gamma_c = \{c' \in \Gamma : c' = \langle Y, r', \bar{s} \rangle \& Y \models \text{Premise}^s(r)\}$$

2. For each case  $c' = \langle Y, r', \bar{s} \rangle$  from  $\Gamma_c$ , let

$$d_{c,c'} = \neg \text{Premise}^{\bar{s}}(r')$$

3. Define

$$D_c = \bigwedge_{c' \in \Gamma_c} d_{c,c'}$$

4. Replace the case  $c = \langle X, r, s \rangle$  from  $\Gamma$  with  $c'' = \langle X, r'', s \rangle$ , where  $r''$  is the new rule

$$\text{Premise}^s(r) \wedge D_c \rightarrow s$$

It is easy to verify that, in each of our three examples, the case base resulting from our sequential rule modification is identical with the case base that would have resulted simply from deciding the same fact situations for the same reasons, and then modifying all rules at once, through refinement. Focusing only on the first of our examples, we can see that  $\Gamma_1' = (\Gamma_1 \cup \{c_3\})^+$ , and then that  $\Gamma_1'' = (\Gamma_1' \cup \{c_4\})^+$ —or, considering the two later decisions together, that  $\Gamma_1'' = (\Gamma_1 \cup \{c_3, c_4\})^+$ .

In these situations, then, where a decision can be accommodated against the background of a case base through an appropriate modification of rules, the same outcome can be achieved, the rules modified in the same way, simply by supplementing the

background case base with that decision and then refining the result. But of course, there are some decisions that cannot be accommodated against the background of certain case bases—the rules simply cannot be modified appropriately. What does refinement lead to in a situation like this? As it turns out, the result of refining the case base supplemented with the new decision will not then be a case base at all, since refinement will produce rules that fail to apply to their corresponding fact situations. And this linkage between accommodation and refinement, I suggest, works in both directions, and can be taken as a formal explication for the concept of accommodation: a decision can be accommodated against the background of a case base just in case the result of supplementing that case base with the decision can itself be refined into a case base.

We can now turn to the notion of precedential constraint itself according to the standard model, in terms of rules that can be modified. The initial idea is that a court is constrained to reach a decision that can be accommodated within the context of a background case base through an appropriate modification of rules—or, given our formal explication of this concept, a decision that can be combined with the background case base to yield a result whose refinement is itself a case base.

**Definition 2 (Rule constraint)** Let  $\Gamma$  be a case base and  $X$  a new fact situation confronting the court. Then the rule constraint requires the court to base its decision on some rule  $r$  leading to an outcome  $s$  such that  $(\Gamma \cup \{(X, r, s)\})^+$  is a case base.

This definition can be illustrated by taking as background the case base  $\Gamma_4 = \{c_9\}$ , containing the single case  $c_9 = \langle X_9, r_9, s_9 \rangle$ , where  $X_9 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$ , where  $r_9 = \{f_1^\pi\} \rightarrow \pi$ , and where  $s_9 = \pi$ . Now suppose the court confronts the new situation  $X_{10} = \{f_1^\pi, f_1^\delta, f_2^\delta, f_3^\delta\}$ , and considers finding for  $\delta$  on the basis of  $f_1^\delta$  and  $f_2^\delta$ , leading to the decision  $c_{10} = \langle X_{10}, r_{10}, s_{10} \rangle$ , where  $X_{10}$  is as above, where  $r_{10} = \{f_1^\delta, f_2^\delta\} \rightarrow \delta$ , and where  $s_{10} = \delta$ . According to current view, this decision is ruled out by precedent, since the result of supplementing the background case base  $\Gamma_4$  with  $c_{10}$  cannot itself be refined into a case base. Indeed, we have  $(\Gamma_4 \cup \{c_{10}\})^+ = \{c_9', c_{10}'\}$  with  $c_9' = \langle X_9', r_9', s_9' \rangle$ , where  $X_9' = X_9$ , where  $r_9' = \{f_1^\pi\} \wedge \neg\{f_1^\delta, f_2^\delta\} \rightarrow \pi$ , and where  $s_9 = \pi$ , and with  $c_{10}' = \langle X_{10}', r_{10}', s_{10}' \rangle$ , where  $X_{10}' = X_{10}$ , where  $r_{10}' = \{f_1^\delta, f_2^\delta\} \wedge \neg\{f_1^\pi\} \rightarrow \delta$ , and where  $s_{10} = \delta$ . But it is easy to see that neither  $c_9'$  nor  $c_{10}'$  is a case, in our technical sense, since the rule  $r_9'$  fails to apply to  $X_9'$ , and the rule  $r_{10}'$  fails to apply to  $X_{10}'$ .

## 4 Constraint by Reasons

Having provided a formal reconstruction of what I take to be the standard account of precedential constraint, in terms of rules that can be modified, I now want to review my own account, developed in terms of an ordering relation on reasons.

In order to motivate this concept, let us return to the case  $c_9 = \langle X_9, r_9, s_9 \rangle$ —where again  $X_9 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$ , where  $r_9 = \{f_1^\pi\} \rightarrow \pi$ , and where  $s_9 = \pi$ —and ask what information is actually carried by this case; what is the court telling

us with its decision? Well, two things, at least. First of all, by appealing to the rule  $r_9$  as justification, the court is telling us that the reason for the decision—that is,  $Premise^\beta(r_9)$ , or  $\{f_1^\pi\}$ —is actually sufficient to justify a decision in favor of  $\pi$ . But second, with its decision for  $\pi$ , the court is also telling us that this reason is preferable to whatever other reasons the case might present that favor the  $\delta$ .

To put this precisely, let us first stipulate that, if  $X$  and  $Y$  are reasons favoring the same side, then  $Y$  is *at least as strong* a reason as  $X$  for that side whenever  $X \subseteq Y$ . Returning to our example, then, where  $X_9 = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta\}$ , it is clear that the strongest reason present for  $\delta$  is  $X_9^\delta = \{f_1^\delta, f_2^\delta\}$ , containing all those factors from the original fact situation that favor  $\delta$ . Since the  $c_9$  court has decided for  $\pi$  on the grounds of the reason  $Premise^\beta(r_9)$ , even in the face of the reason  $X_9^\delta$ , it seems to follow as a consequence of the court's decision that the reason  $Premise^\beta(r_9)$  for  $\pi$  is preferred to the reason  $X_9^\delta$  for the  $\delta$ —that is, that  $\{f_1^\pi\}$  is preferred to the reason  $\{f_1^\delta, f_2^\delta\}$ . If we introduce the symbol  $<_{c_9}$  to represent the preference relation on reasons that is derived from the particular case  $c_9$ , then this consequence of the court's decision can be put more formally as the claim that  $\{f_1^\delta, f_2^\delta\} <_{c_9} \{f_1^\pi\}$ , or equivalently, that  $X_9^\delta <_{c_9} Premise^\beta(r_9)$ .

As far as the preference ordering goes, then, the earlier court is telling us at least that  $X_9^\delta <_{c_9} Premise^\beta(r_9)$ , but is it telling us anything else? Perhaps not explicitly, but implicitly, yes. For if the reason  $Premise^\beta(r_9)$  for  $\pi$  is preferred to the reason  $X_9^\delta$  for  $\delta$ , then surely any reason for  $\pi$  that is at least as strong as  $Premise^\beta(r_9)$  must likewise be preferred to  $X_9^\delta$ , and just as surely,  $Premise^\beta(r_9)$  must be preferred to any reason for  $\delta$  that is at least as weak as  $X_9^\delta$ . As we have seen, a reason  $Z$  for  $\pi$  is at least as strong as  $Premise^\beta(r_9)$  if it contains all the factors contained by  $Premise^\beta(r_9)$ —that is, if  $Premise^\beta(r_9) \subseteq Z$ . And we can conclude, likewise, that a reason  $W$  for  $\delta$  is at least as weak as  $X_9^\delta$  if it contains no more factors than  $X_9^\delta$  itself—that is, if  $W \subseteq X_9^\delta$ . It therefore follows from the earlier court's decision in  $c_9$ , not only that  $X_9^\delta <_{c_9} Premise^\beta(r_9)$ , but that  $W <_{c_9} Z$  whenever  $W$  is at least as weak a reason for  $\delta$  as  $X_9^\delta$  and  $Z$  is at least as strong a reason for  $\pi$  as  $Premise^\beta(r_9)$ —whenever, that is,  $W \subseteq X_9^\delta$  and  $Premise^\beta(r_9) \subseteq Z$ . To illustrate: from the court's explicit decision that  $\{f_1^\delta, f_2^\delta\} <_{c_9} \{f_1^\pi\}$ , we can conclude also that  $\{f_1^\delta\} <_{c_9} \{f_1^\pi, f_3^\pi\}$ , for example.

This line of argument leads to the following definition of the preference relation among reasons that can be derived from a single case.

**Definition 3 (Preference relation derived from a case)** Let  $c = \langle X, r, s \rangle$  be a case, and suppose  $W$  and  $Z$  are reasons. Then the relation  $<_c$  representing the preferences on reasons derived from the case  $c$  is defined by stipulating that  $W <_c Z$  if and only if  $W \subseteq X^s$  and  $Premise^s(r) \subseteq Z$ .

Once we have defined the preference relation derived from a single case, we can introduce a preference relation  $<_\Gamma$  derived from an entire case base  $\Gamma$  in the natural way, by stipulating that one reason is stronger than another according to the entire case base if that strength relation is supported by some particular case in the case base.

**Definition 4 (Preference relation derived from a case base)** Let  $\Gamma$  be a case base, and suppose  $W$  and  $Z$  are reasons. Then the relation  $<_{\Gamma}$  representing the preferences on reasons derived from the case base  $\Gamma$  is defined by stipulating that  $W <_{\Gamma} Z$  if and only if  $W <_c Z$  for some case  $c$  from  $\Gamma$ .

And we can then define a case base as inconsistent if it provides conflicting information about the preference relation among reasons—telling us, for any two reasons, the each is preferred to the other—and consistent otherwise.

**Definition 5 (Reason consistent case bases)** Let  $\Gamma$  be a case base with  $<_{\Gamma}$  the derived preference relation. Then  $\Gamma$  is reason inconsistent if and only if there are reasons  $X$  and  $Y$  such that  $X <_{\Gamma} Y$  and  $Y <_{\Gamma} X$ .  $\Gamma$  is reason consistent if and only if it is not reason inconsistent.

Given this notion of consistency, we can now turn to the concept of precedential constraint itself, according to the reason account. The intuition could not be simpler: in deciding a case, a constrained court is required to preserve the consistency of the background case base. Suppose, more exactly, that a court constrained by a background case base  $\Gamma$  is confronted with a new fact situation  $X$ . Then the court is required to reach a decision on  $X$  that is itself consistent with  $\Gamma$ —that is, a decision that does not result in an inconsistent case base.

**Definition 6 (Reason constraint)** Let  $\Gamma$  be a case base and  $X$  a new fact situation confronting the court. Then reason constraint requires the court to base its decision on some rule  $r$  leading to an outcome  $s$  such that the new case base  $\Gamma \cup \{(X, r, s)\}$  is reason consistent.

This idea can be illustrated by assuming as background the previous case base  $\Gamma_4 = \{c_9\}$ , containing only the previous case  $c_9$ , supposing once again that, against this background, the court confronts the fresh situation  $X_{10} = \{f_1^{\pi}, f_1^{\delta}, f_2^{\delta}, f_3^{\delta}\}$  and considers finding for  $\delta$  on the basis of  $f_1^{\delta}$  and  $f_2^{\delta}$ , leading to the decision  $c_{10} = \langle X_{10}, r_{10}, s_{10} \rangle$ , where  $X_{10}$  is as above, where  $r_{10} = \{f_1^{\delta}, f_2^{\delta}\} \rightarrow \delta$ , and where  $s_{10} = \delta$ . We saw in the previous section that such a decision would fail to satisfy the rule constraint, and we can see now that it fails to satisfy the reason constraint as well. Why? Because the new case  $c_{10}$  would support the preference relation  $\{f_1^{\pi}\} <_{c_{10}} \{f_1^{\delta}, f_2^{\delta}\}$ , telling us that the reason  $\{f_1^{\delta}, f_2^{\delta}\}$  for  $\delta$  outweighs the reason  $\{f_1^{\pi}\}$  for  $\pi$ . But  $\Gamma_4$  already contains the case  $c_9$ , from which we can derive the preference relation  $\{f_1^{\delta}, f_2^{\delta}\} <_{c_9} \{f_1^{\pi}\}$ , telling us exactly the opposite. As a result, the augmented case base  $\Gamma_4 \cup \{c_{10}\}$  would be reason inconsistent.

## 5 An Equivalence

The two accounts presented in Sects. 3 and 4 of this paper offer strikingly different pictures of precedential constraint, and of legal development and norm change. According to the standard account from Sect. 3, what is important about a background case base is the set of rules it contains, together with the facts of the cases in



which they were formulated. In reaching a decision concerning a new fact situation, the court is obliged by modify the existing set of rules appropriately, to accommodate this decision. Precedential constraint derives from the fact that such accommodation is not always possible; legal development is due to the modification of existing rules, together with the addition to the case base of the new rule from the new decision. According to the reason account from Sect. 4, what is important about a background case base is, not the set of rules it contains, but a derived preference ordering on reasons. In confronting a new fact situation, then, a court is obliged only to reach a decision that is consistent with the existing preference ordering on reasons. Constraint derives from the fact that not all such decisions are consistent; legal development is due to the supplementation of the existing preference ordering on reasons with the new preferences derived from the new decision.

Given the very different pictures presented by these two accounts of precedential constraint, it is interesting to note that the accounts are in fact equivalent, in the sense that, given a background case base  $\Gamma$  and a new fact situation  $X$ , a decision on the basis of a rule  $r$  is permitted by the rule constraint just in case it is permitted by the reason constraint. This observation—the chief result of the paper—can be established very simply, after showing, first, that any reason consistent case base has a case base as its refinement, and second, that any case base with a case base as its refinement must be reason consistent.

**Observation 1** If  $\Gamma$  is a reason consistent case base, then its refinement  $\Gamma^+$  is a case base.

**Proof** Suppose  $\Gamma$  is a reason consistent case base.  $\Gamma^+$  is constructed from  $\Gamma$  by replacing each case  $c = \langle X, r, s \rangle$  from  $\Gamma$  with the new  $c'' = \langle X, r'', s \rangle$ , where the new rule  $r''$  has the form  $Premise^s(r) \wedge D_c \rightarrow s$ , as specified as in Definition 1. Since all of the new rules involved in moving from  $\Gamma$  to  $\Gamma^+$  support the same outcomes as the original, we can verify that  $\Gamma^+$  is a case base as well simply by establishing that, for each  $c'' = \langle X, r'', s \rangle$  from  $\Gamma^+$ , the new rule  $r''$  continues to be applicable to the fact situation  $X$ —that is, that  $X \models Premise(r'')$ , or that  $X \models Premise^s(r) \wedge D_c$ . We know, of course, that  $X \models Premise^s(r)$ , since  $\Gamma$  is a case base, and so need only show that  $X \models D_c$ .

It follows from Steps 2 and 3 of the construction that establishing that  $X \models D_c$  amounts to showing, for each  $c' = \langle Y, r', \bar{s} \rangle$  from  $\Gamma_c$ , where  $c = \langle X, r, s \rangle$ , that  $X \models \neg Premise^{\bar{s}}(r')$ . So suppose the contrary—that  $X \not\models \neg Premise^{\bar{s}}(r')$ , or  $X \models Premise^{\bar{s}}(r')$ , from which we can conclude that (1)  $Premise^{\bar{s}}(r') \subseteq X^{\bar{s}}$ . Since  $c' = \langle Y, r', \bar{s} \rangle$  belongs to  $\Gamma_c$ , we know from Step 1 of the construction that  $Y \models Premise^s(r)$ , from which we can conclude that (2)  $Premise^s(r) \subseteq Y^s$ . From (1), we can then conclude by Definition 3 that (3)  $Premise^{\bar{s}}(r') <_c Premise^s(r)$ , and from (2), that (4)  $Premise^s(r) <_{c'} Premise^{\bar{s}}(r')$ . But since both  $c$  and  $c'$  belong to  $\Gamma$ , the combination of (3) and (4) contradicts the stipulation that  $\Gamma$  is reason consistent. Hence, our assumption fails, from which we can conclude that  $X \models D_c$ . ■

**Observation 2** If  $\Gamma$  is a case base whose refinement  $\Gamma^+$  is also a case base, then  $\Gamma$  is reason consistent.



**Proof** Suppose  $\Gamma$  is a case base whose refinement  $\Gamma^+$  is a case base, but that  $\Gamma$  itself is not reason consistent. Since  $\Gamma$  is not reason consistent, there are reasons  $A$  and  $B$  such that (1)  $A <_c B$  and (2)  $B <_{c'} A$  for cases  $c = \langle X, r, s \rangle$  and  $c' = \langle Y, r', \bar{s} \rangle$  from  $\Gamma$ . From (1) we have (3)  $A \subseteq X^s$  and (4)  $Premise^s(r) \subseteq B$ , and from (2) we have (5)  $B \subseteq Y^{\bar{s}}$  and (6)  $Premise^{\bar{s}}(r') \subseteq A$ . Together, (4) and (5), along with the fact that  $Y^s \subseteq Y$ , yield  $Premise^s(r) \subseteq Y$ , or (7)  $Y \models Premise^s(r)$ . In the same way, (3) and (6), together with the fact that  $X^{\bar{s}} \subseteq X$ , yield  $Premise^{\bar{s}}(r') \subseteq X$ , or (8)  $X \models Premise^{\bar{s}}(r')$ .

$\Gamma^+$  is constructed from the case base  $\Gamma$  by replacing each case  $c = \langle X, r, s \rangle$  with the new  $c'' = \langle X, r'', s \rangle$ , where the new rule  $r''$  has the form  $Premise^s(r) \wedge D_c \rightarrow s$ , as specified in Definition 1. Step 1 of this construction, together with (7), tells us that  $c'$  belongs to  $\Gamma_c$ , and then Steps 2, 3, and 4 allow us to conclude that  $\neg Premise^{\bar{s}}(r')$  is one of the conjuncts of  $D_c$ , and so of the new rule  $r''$ . From (8), however, we know that  $X \models Premise^{\bar{s}}(r')$ , from which it follows that  $X \not\models \neg Premise^{\bar{s}}(r')$ . As a result, the rule of  $c''$  does not apply to its facts, from which it follows that  $c''$  is not a case, and so  $\Gamma^+$  not a case base, contrary to our assumption. ■

**Observation 3** Let  $\Gamma$  be a case base, and let  $X$  be a new fact situation. Then a decision on the basis of a rule  $r$  leading to an outcome  $s$  is permitted by the reason constraint just in case that decision is permitted by the rule constraint.

**Proof** Suppose a decision on the basis of  $r$  and leading to the outcome  $s$  is permitted by the reason constraint, so that  $\Gamma \cup \{\langle X, r, s \rangle\}$  is reason consistent. Then  $(\Gamma \cup \{\langle X, r, s \rangle\})^+$  is a rule coherent case base, by Observation 1, so that the same decision is permitted by the rule constraint. Or suppose a decision on the basis of  $r$  and leading to the outcome  $s$  is permitted by the rule constraint, so that  $(\Gamma \cup \{\langle X, r, s \rangle\})^+$  is a case base. Then  $\Gamma \cup \{\langle X, r, s \rangle\}$  is reason consistent by Observation 2, so that the same decision is permitted by the reason constraint. ■

## 6 Discussion

The goal of this paper has been to establish the equivalence between two accounts of precedential constraint, the standard account from Sect. 3 and the reason account from Sect. 4. I discuss what I take to be the philosophical significance of this equivalence elsewhere.<sup>12</sup> Here I simply want to close with two technical remarks, one concerning the standard account and one concerning the reason account.

Beginning with the standard account, developed in terms of rules that can be modified, it is natural to ask *why* these rules should be modified—what property of the overall case base can we suppose courts are trying to establish, or guarantee, through the modification of rules? That natural answer to this natural question is that, by modifying rules, courts are trying to guarantee a kind of consistency. More exactly, suppose we take

<sup>12</sup> See Horty (2013).



$$Rule(\Gamma) = \{Rule(c) : c \in \Gamma\}$$

as the set of rules derived from a case base  $\Gamma$ . We can then define a case base as rule consistent just in case, whenever a rule derived from that case base applies to the facts of some case from the case base, the rule yields an outcome identical to the outcome that was actually reached in that case.

**Definition 7 (Rule consistent case base)** A case base  $\Gamma$  is rule consistent if and only if, for each  $r$  in  $Rule(\Gamma)$  and for each  $c$  in  $\Gamma$  such that  $Factors(c) \models Premise(r)$ , we have  $Outcome(c) = Conclusion(r)$ .

And it is natural to conjecture that, by modifying rules to accommodate later decisions, the property that courts are trying to preserve is the property of rule consistency.

This conjecture is supported by reflection on the examples we used to motivate the standard account. Recall our initial example. Here, the background case base was  $\Gamma_1 = \{c_2\}$ , with  $c_2 = \langle X_2, r_2, s_2 \rangle$ , where  $X_2 = \{f_1^\pi, f_1^\delta\}$ , where  $r_2 = \{f_1^\pi\} \rightarrow \pi$ , and where  $s_2 = \pi$ ; and we imagined that the court, confronting the new fact situation  $X_3 = \{f_1^\pi, f_2^\delta\}$ , wishes to decide for  $\delta$  on the basis of  $f_2^\delta$ , leading to the decision  $c_3 = \langle X_3, r_3, s_3 \rangle$ , where  $X_3$  is as above, where  $r_3 = \{f_2^\delta\} \rightarrow \delta$ , and where  $s_3 = \delta$ . Now suppose the rule from the original case had not been modified, so that the result of this decision was that the original case base was simply supplemented with the new decision, leading to the revised case base  $\Gamma_1 \cup \{c_3\}$ . It is easy to see that the revised case base would not be rule consistent, since  $r_2$  belongs to  $Rule(\Gamma_1 \cup \{c_3\})$  and  $Factors(c_3) \models Premise(r_2)$ , yet  $Outcome(c_3) \neq Conclusion(r_2)$ —the rule from the original case applies to the facts of the new case, but supports a different result from that actually reached in the new case. By modifying the original rule  $r_2$  to have the force of  $r_2' = \{f_1^\pi\} \wedge \neg\{f_2^\delta\} \rightarrow \pi$ , the court can thus be seen as guaranteeing rule consistency, blocking application of the rule to a case with a conflicting outcome.

Our other motivating examples have the same form: the later decisions would introduce rule inconsistency on their own, but modification of the earlier rules restores consistency. Is it, then, rule consistency that we should see a court as attempting to guarantee by modifying rules? Surprisingly, perhaps, I would say No—for some case bases are peculiar even though they are rule consistent. Consider, for example, the case base  $\Gamma_5 = \{c_{11}\}$  with  $c_{11} = \langle X_{11}, r_{11}, s_{11} \rangle$ , where  $X_{11} = \{f_1^\pi, f_2^\pi, f_1^\delta\}$ , where  $r_{11} = \{f_1^\pi\} \rightarrow \pi$ , and where  $s_{11} = \pi$ . And suppose that, against the background of this case base, the court confronts the new fact situation  $X_{12} = \{f_1^\pi, f_1^\delta, f_2^\delta\}$  and wishes to decide for  $\delta$  on the basis of  $f_1^\delta$ . There is, then, the risk of rule inconsistency, since the previous rule  $r_{11}$  applies to the fact situation  $X_{12}$ , but leads to  $\pi$  as an outcome, rather than  $\delta$ . But now, imagine that the court distinguishes in the following way: first, by modifying the previous  $r_{11}$  to have the force of  $r_{11}' = \{f_1^\pi\} \wedge \neg\{f_2^\delta\} \rightarrow \pi$ , and second, by hedging its rule for the new case to read  $r_{12} = \{f_1^\delta\} \wedge \neg\{f_2^\pi\} \rightarrow \delta$ . The resulting case base would then be  $\Gamma_5' = \{c_{11}', c_{12}\}$ , with  $c_{11}' = \langle X_{11}', r_{11}', s_{11}' \rangle$ , where  $X_{11}' = X_{11}$ , where  $r_{11}'$  is as above, and where  $s_{11}' = s_{11}$ , and with

$c_{12} = \langle X_{12}, r_{12}, s_{12} \rangle$ , where  $X_{12} = \{f_1^\pi, f_1^\delta, f_2^\delta\}$ , where  $r_{12} = \{f_1^\delta\} \wedge \neg\{f_2^\pi\} \rightarrow \delta$ , and where  $s_{12} = \delta$ .

It is easy to see that the new case base  $\Gamma_5'$  is rule consistent, since neither of the rules involved applies to the other case. It is, nevertheless, a peculiar case base. One way of seeing this is by noting that, although rule consistent, the case base is not reason consistent: we have both  $\{f_1^\delta\} <_{c_{11}'} \{f_1^\pi\}$  and  $\{f_1^\pi\} <_{c_{12}} \{f_1^\delta\}$ . Another way—which gets to the root of the problem—is by noting that, in each of the two case rules, the exception clause, which blocks applicability to the other case, has nothing to do with the reason for which that other case was decided.

Because a case base can be peculiar even if it is rule consistent, I do not think that mere rule consistency is the property that courts are concerned to guarantee, as they modify rules. Instead, I believe, courts must be seen as trying to avoid, not just rule inconsistency, but also peculiarity in the sense illustrated above, by guaranteeing the property of rule coherence.

**Definition 8 (Rule coherent case base)** Let  $\Gamma$  be a case base. Then  $\Gamma$  is rule coherent just in case, for each  $c = \langle X, r, s \rangle$  and  $c' = \langle Y, r', \bar{s} \rangle$  in  $\Gamma$ , if  $Y \models \text{Premise}^s(r)$ , then  $\text{Premise}(r) \vdash \neg \text{Premise}^{\bar{s}}(r')$ .

What this property requires is that, whenever the reason for a decision in some particular case holds in another case where the opposite outcome was reached, then the negation of the reason for the latter decision must be entailed by the premise of the rule supporting the original.<sup>13</sup> The property of rule coherence is thus supposed to be explanatory in a way that mere rule consistency is not: when the original reason holds in a latter case but fails to yield the appropriate outcome, the rule putting forth the original reason must help us understand why, by containing the information that it does not apply when the reason from the latter case is present.

We can verify that rule coherence is a stronger property than mere rule consistency, in the sense that a rule coherent case must be rule consistent.

**Observation 4** Any rule coherent case base is rule consistent.

**Proof** Suppose a case base  $\Gamma$  is rule coherent but not rule consistent. Since  $\Gamma$  is not rule consistent,  $\text{Rule}(\Gamma)$  contains some rule  $r$ , derived from some case  $c = \langle X, r, s \rangle$  belonging to  $\Gamma$ , for which there is another case  $c' = \langle Y, r', \bar{s} \rangle$  from  $\Gamma$  such that  $Y \models \text{Premise}(r)$  and  $\text{Conclusion}(r) \neq \text{Outcome}(c)$ . Since  $Y \models \text{Premise}(r)$ , we know that  $Y \models \text{Premise}^s(r)$ , of course. By rule coherence, we then have  $\text{Premise}(r) \vdash \neg \text{Premise}^{\bar{s}}(r')$ , from which it follows that  $Y \not\models \text{Premise}^{\bar{s}}(r')$ , so that  $Y \not\models \text{Premise}(r')$ , which contradicts the requirement that the rule of  $c' = \langle Y, r', \bar{s} \rangle$  must be applicable to the facts of the case. ■

And we can also see that, in contrast with case bases that are merely rule consistent, a case base that is rule coherent must be reason consistent as well.

<sup>13</sup> The most straightforward way in which the negation of the reason for the opposite decision would be entailed by the rule supporting the original, of course, is by being contained explicitly among the exceptions to that rule; but speaking more generally of entailment allows for other encodings as well.

**Observation 5** Any rule coherent case base is reason consistent.

**Proof** Suppose a case base  $\Gamma$  is rule coherent but not reason consistent. Since  $\Gamma$  is not reason consistent, there are reasons  $A$  and  $B$  such that (1)  $A <_c B$  and (2)  $B <_{c'} A$  for cases  $c = \langle X, r, s \rangle$  and  $c' = \langle Y, r', \bar{s} \rangle$  from  $\Gamma$ . From (1) we have (3)  $A \subseteq X^{\bar{s}}$  and (4)  $\text{Premise}^s(r) \subseteq B$ , and from (2) we have (5)  $B \subseteq Y^s$  and (6)  $\text{Premise}^{\bar{s}}(r') \subseteq A$ . Together, (4) and (5), along with the fact that  $Y^s \subseteq Y$ , yield  $\text{Premise}^s(r) \subseteq Y$ , or (7)  $Y \models \text{Premise}^s(r)$ . In the same way, (3) and (6), together with the fact that  $X^{\bar{s}} \subseteq X$ , yield  $\text{Premise}^{\bar{s}}(r') \subseteq X$ , or (8)  $X \models \text{Premise}^{\bar{s}}(r')$ . From (7), rule coherence tells us that  $\text{Premise}(r) \vdash \neg \text{Premise}^{\bar{s}}(r')$ , or that (9)  $\text{Premise}^{\bar{s}}(r') \vdash \neg \text{Premise}(r)$ . But then (8) and (9) tell us that  $X \models \neg \text{Premise}(r)$ , or that (10)  $X \not\models \text{Premise}(r)$ , which contradicts the requirement that the rule of  $c = \langle X, r, s \rangle$  must be applicable to the facts of that case. ■

It follows from this last observation that, unlike case bases in general, any case base that is rule coherent must have a case base as its refinement. Why is this? Because the observation tells us that any rule coherent case base is reason consistent, and we know from Observation 1 that the refinement of a reason consistent case base is a case base. Of course, a case base might well be reason consistent without being rule coherent—though the case base is reason consistent, its rules may simply not have been modified properly. But it is easy to see that, once the rules of a reason consistent case base have been modified through refinement, the result will be a rule coherent case base.

**Observation 6** The refinement of a reason consistent case base is a rule coherent case base.

**Proof** The proof of Observation 1 shows that the refinement  $\Gamma^+$  of a reason consistent case base  $\Gamma$  is a case base. To see that  $\Gamma^+$  is also rule coherent, we need only continue that proof by noting that, where  $c'' = \langle X, r'', s \rangle$  is the new case replacing the original case  $c = \langle X, r, s \rangle$  from  $\Gamma$ , it follows from the construction of  $\Gamma^+$  that  $\text{Premise}^s(r'')$  is identical with  $\text{Premise}^s(r)$ , so that, for any case  $c' = \langle Y, r', \bar{s} \rangle$  from  $\Gamma^+$ , whenever  $Y \models \text{Premise}^s(r'')$ , the formula  $\neg \text{Premise}^{\bar{s}}(r')$  is a conjunct of  $\text{Premise}(r'')$ . From this we can conclude that  $\text{Premise}(r'') \vdash \neg \text{Premise}^{\bar{s}}(r')$  at once. ■

Turning now from the standard account to the reason account, it is worth noting that our definition of reason constraint makes sense only on the assumption that the background case base is itself consistent to begin with. This is, of course, an unrealistic assumption. Given the vagaries of judicial decision, with a body of case law developed by a number of different courts, at different places and different times, it would be surprising if any nontrivial case base were actually consistent. But in fact, this assumption is not essential. The notion of reason inconsistency at work here is not like logical inconsistency—it is local, not pervasive. A case base might be reason inconsistent in certain areas, providing conflicting information about the relative priority of particular reasons, while remaining consistent elsewhere. It is therefore possible to extend our account of reason constraint to apply also to inconsistent

case bases, by requiring of a court, not necessarily that its decision should not yield an inconsistent case base, but only that its decision should not introduce any new inconsistencies, which were not present before, into a case base that may already be inconsistent.

To state this precisely, we can define an *inconsistency* in a case base  $\Gamma$  as a pair of reasons,  $Y$  and  $Z$ , such that  $Y <_{\Gamma} Z$  and  $Z <_{\Gamma} Y$ . The idea that a court should introduce no new inconsistencies into a case base can then be captured through the requirement that every inconsistency present after the court's decision must already have been present prior to the decision, leading to the following definition.

**Definition 9 (Reason constraint: general version)** Let  $\Gamma$  be a case base and  $X$  a new fact situation confronting the court. Then reason constraint requires the court to base its decision on some rule  $r$  leading to an outcome  $s$  such that: whenever  $Y <_{\Gamma \cup \{(X,r,s)\}} Z$  and  $Z <_{\Gamma \cup \{(X,r,s)\}} Y$ , we also have  $Y <_{\Gamma} Z$  and  $Z <_{\Gamma} Y$ .

This more general definition of reason constraint can be illustrated by considering the case base  $\Gamma_6 = \{c_{13}, c_{14}\}$ , with  $c_{13} = \langle X_{13}, r_{13}, s_{13} \rangle$ , where  $X_{13} = \{f_1^{\pi}, f_1^{\delta}, f_2^{\delta}\}$ , where  $r_{13} = \{f_1^{\pi}\} \rightarrow \pi$ , and where  $s_{13} = \pi$ , and with  $c_{14} = \langle X_{14}, r_{14}, s_{14} \rangle$ , where  $X_{14} = \{f_1^{\pi}, f_1^{\delta}\}$ , where  $r_{14} = \{f_1^{\delta}\} \rightarrow \delta$ , and where  $s_{14} = \delta$ . This case base is inconsistent, of course, since it tells us both that  $\{f_1^{\delta}\} <_{\Gamma_6} \{f_1^{\pi}\}$  and that  $\{f_1^{\pi}\} <_{\Gamma_6} \{f_1^{\delta}\}$ . But now, suppose that, against the background of this case base, the court confronts the new fact situation  $X_{15} = \{f_1^{\pi}, f_2^{\delta}\}$ . According to our original Definition 6, nothing the court can do is right, since the case base is already inconsistent. According to our more general definition, however, there is nevertheless a right decision for the court to make, even though the background case base is inconsistent, and a wrong decision. The right decision in this new situation would be to find for  $\pi$  on the basis of  $f_1^{\pi}$ , since this introduces no new inconsistencies. The wrong decision would be to find for  $\delta$  on the basis of  $f_2^{\delta}$ , leading to the new case base  $\Gamma_6 \cup \{c_{15}\}$ , with  $c_{15} = \langle X_{15}, r_{15}, s_{15} \rangle$ , where  $X_{15} = \{f_1^{\pi}, f_2^{\delta}\}$ , where  $r_{15} = \{f_2^{\delta}\} \rightarrow \delta$ , and where  $s_{15} = \delta$ . This decision would introduce a new inconsistency, since we would then have both  $\{f_2^{\delta}\} <_{\Gamma_6 \cup \{c_{15}\}} \{f_1^{\pi}\}$  and  $\{f_1^{\pi}\} <_{\Gamma_6 \cup \{c_{15}\}} \{f_2^{\delta}\}$ , even though we did not previously have both  $\{f_2^{\delta}\} <_{\Gamma_6} \{f_1^{\pi}\}$  and  $\{f_1^{\pi}\} <_{\Gamma_6} \{f_2^{\delta}\}$ .

## 7 Conclusion

There are many other issues to explore, both technical and philosophical. The case-based priority ordering on reasons is not transitive. Of course, we could simply impose transitivity, by reasoning with the transitive closure of the basic relation; but the question of whether we should leads to a thicket of interesting problems concerning belief combination. In addition, the rules we work with are very simple in form—basically, nothing but a reason supporting a conclusion, and a list of contrary reasons that are required not to hold, if that conclusion is to be reached. Should we allow more complex rules, and if so, how complex? This question, likewise, has

a technical side, concerning the ways in which more complex rules might be unwound into ordering relations on reasons, and a conceptual side, since in the common law, courts are not supposed to legislate, but simply to respond to the fact situations before them. How complex can the rules become before we are forced to say that courts are legislating, rather than ruling on cases?

These questions, and others, will have to wait for another occasion. The present paper has a more limited aim. The very influential work by David Makinson and his AGM collaborators began with reflections, by Alchourrón and Makinson, on norm change in the civil law—and my goal here has been simply to sketch one way in which a complimentary theory of norm change in the common law might be developed. In doing so, I find that the line of thought traced here conforms to several, though not all, of the maxims set out by David in a later article, in which he tries to explain what was different about the initial work in the AGM tradition.<sup>14</sup> The relevant maxims are:

*Logic is not just about deduction*  
*There is nothing wrong with classical logic*  
*Don't internalize too quickly*  
*Do some logic without logic*

Concerning the first of these maxims, it was a notable feature of AGM that, while the overwhelming majority of contemporary philosophical logicians were exploring different consequence relations—extensions of or alternatives to classical logic—the authors of that work focused on the entirely separate topic of belief revision; the present paper, likewise, applies logical techniques to a topic other than the question of what follows from what. Concerning the second maxim, in moving into a new field, the AGM authors carried with them the familiar classical logic; and likewise the present paper. The point of the third maxim is that it is often useful to explore certain concepts in the metalanguage, before trying to represent these concepts through an explicit object language connective, not for Quinean or other philosophical reasons, but simply as a matter of research methodology—get the flat case right, before considering nesting or iteration. This maxim has particular relevance for the present work, since I had originally thought it best, in exploring precedential constraint, to concentrate on the obligations of later courts, with these obligations represented through an object-language deontic operator; it was only when that operator was removed that the shape of the current approach became clear. Finally, as to the fourth maxim, while there are some connectives in the present account—conjunction, negation—it should be clear that they are not contributing much: there is no nesting of formulas, for example. Just as with AGM, while there may be connectives present, the real interest lies elsewhere.

To these four maxims, I would add a fifth:

*Sometimes, let the subject shape the logic*

One frequently finds that a theorist brings an existing logic or set of logical techniques, particularly those already familiar to the theorist, to a new subject. There is nothing

<sup>14</sup> Makinson (2003).



wrong with that: it is important to develop existing theories, and to explore new applications. But sometimes, existing techniques do not fit the new subject, or do not fit it well. Then there is the opportunity to let the subject suggest new logical ideas, and it is important to be open to that opportunity. This is a path that David has taken more than once—not only in his AGM work, but also in other work that I am familiar with, such as his research on Hohfeld's rights relations, on the general theory of nonmonotonic consequence, on norms without truth values, and on input/output logics.<sup>15</sup> It is a path that has already led him to many vital contributions, and we can expect that it will lead to many more.

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<sup>15</sup> See Makinson (1986, 1989, 1994, 1998) and Makinson and van der Torre (2000).



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